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ABSTRACT

The Texas Educational Assessment of Minimum Skills (TEAMS) was developed by the Texas Education Agency (TEA), in response to Texas House Bill 72 (1984), which mandated a new basic skills assessment program for mathematics, reading, and writing at grades 1, 3, 5, 7, 9 and 11. The test given to students in Grade 11 is an exit-level examination, and Texas students must pass this test prior to receiving their high school diploma. This document is one in a series of Instructional Strategies Guides designed to familiarize Texas educators with the types of strategies employed in assessing learner mastery of TEAMS objectives and to provide suggested instructional activities that can be used to promote mastery of those objectives. This guide focuses on the mathematics objecives measured on the TEAMS Exit Level Test. It provides a detailed description of the test items used to measure each objective. The 18 objectives are subsumed under 7 skill areas: (1) number concepts (number sequence, rounding, equivalencies, and exponential notation); (2) computation (fractions and mixed numbers, decimals, and integers); (3) problem solving (multiple operations, proportions, and percents); (4) measurement; (5) geometry (geometric formulas and properties of geometric shapes); (6) probability, statistics and graphing (averages, simple probability, graphs and tables); and (7) algebraic concepts (formulas, solve for a variable). The following information is provided for each objective: (1) a description of the objective; (2) a sample item; (3) test questions; (4) answer choices; (5) answer analysis for the sample item; (6) instructional emphases; and (7) possible instructional activities. (JAZ)

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ED277759

TEAMS

INSTRUCTIONAL STRATEGIES GUIDE

MATHEMATICS
EXIT LEVEL

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1986

TEAMS INSTRUCTIONAL STRATEGIES GUIDE

MATHEMATICS

EXIT LEVEL

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INTRODUCTION

During the period 1980-1985 Texas had a criterion-referenced testing program designed to measure minimum basic skills in mathematics, reading, and writing. That program was known as the Texas Assessment of Basic Skills (TABS).

Beginning with the 1985-1986 school year, a new assessment program, the Texas Educational Assessment of Minimum Skills (TEAMS), was instituted to measure minimum basic skills in mathematics, reading, and writing in Grades 1, 3, 5, 7, 9, and 11. The test given to students in Grade 11 is an exit-level examination and must be passed by Texas students prior to their being awarded a high school diploma. This assessment program was mandated by House Bill 72, passed by the Second Called Session of the 68th Texas Legislature in July 1984.

To implement the TEAMS testing program, an extensive developmental phase was begun during the 1984-1985 school year. Basic skills objectives were selected after a thorough review of the State Board of Education Rules for Curriculum, GE4 450 01, (Essential Elements). Measurement specifications were written, test items were developed, and field testing was conducted. School district personnel from across the state were active participants in all aspects of the program. The TEAMS tests were approved by the State Board of Education.

Because of the significance of the TEAMS examinations, TEA officials decided to provide Texas educators with instructional strategies guides dealing with those examinations. These guides were designed to provide (1) a clear description of the way in which TEAMS objectives are assessed plus (2) instructional considerations and activities that might be used by Texas educators in designing instructional strategies to promote mastery of TEAMS objectives. This instructional strategies guide focuses on the mathematics objectives measured on the TEAMS Exit Level Test.

For each mathematics objective assessed in the TEAMS Exit Level Test, a detailed description of the kinds of test items used to measure that objective has been provided. In addition, for each objective a brief instructional analysis has been provided along with possible instructional activities that might be employed to promote student mastery of the objective.

It must be stressed that the instructionally related components of this guide are in no way intended to be prescriptive. Texas educators will vary substantially in the ways that they promote learner mastery of the TEAMS objectives. The segments of the guide which deal with instructional possibilities should be viewed, therefore, only as potential approaches to the TEAMS objectives.

Components of the Instructional Strategies Guide

Eighteen objectives are measured on the TEAMS Exit Level Test. Each of these objectives has been categorized into one of seven skill areas. The distribution of the mathematics objectives per skill area is shown below:

SKILL AREA	OBJECTIVE
Number Concepts	Number Sequence Rounding Equivalencies Exponential Notation
Computation	Fractions and Mixed Numbers Decimals Integers

Problem Solving	Multiple Operations Proportions Percents
Measurement	Measurement
Geometry	Geometric Formulas Properties of Geometric Shapes
Probability, Statistics, and Graphing	Averages Simple Probability Graphs and Tables
Algebraic Concepts	Formulas Solve for a Variable

For each of the objectives included in the mathematics section of the TEAMS Exit Level Test, a set of specific components has been provided in the guide. Each of these components will be briefly described.

Description of the objective. A short description of an objective initiates consideration of that objective. Typically presented in one or two sentences, this description provides a succinct overview of the objective's focus.

Sample item. Following the description of each objective, an illustrative sample item is provided. This item was constructed according to the test specifications for the objective, hence is representative of the TEAMS test items which will be used to measure students' mastery of the objective. By reviewing the sample item along with the brief description of the objective, one can ascertain the general nature of the objective and how it is assessed.

Test questions. To provide a more detailed view of how each TEAMS objective is assessed, the next set of descriptive information deals with the "stimulus" segment of each test item, that is, an item's test question. The most important elements in the item's test question will be listed.

Answer choices. The next section of descriptive information deals with the "response" segment of each test item, that is, an item's answer choices. The most important characteristics of an item's answer choices will be described. For instance, in addition to noting that there will be one correct answer choice, the descriptive information will specify the categories of incorrect answer choices. All incorrect answer choices will fall into one of these categories.

Answer analysis for sample item. After the description of answer choices in general terms, there will be a specific analysis of the answer choices from the sample item. This analysis will explain why the correct answer is right and why the incorrect answers are wrong.

Instructional emphases. An effort has been made to describe one or more instructional considerations relevant to the design of instructional sequences for each TEAMS objective. These instructional analyses are not intended to represent the only important points associated with instruction aimed at a particular objective. Yet, it is hoped that an educator's thinking about a given objective might be stimulated as a consequence of these brief discussions.

Possible instructional activities. The final set of materials for each TEAMS objective consists of several possible instructional activities that Texas educators may wish to consider as they organize their instructional plans. For ease of identification, each activity has been given a name. You may find that many of the activities can be adapted with only modest changes to provide practice in a variety of skill areas.

Use of the Instructional Strategies Guide

It is hoped that this TEAMS Instructional Strategies Guide will accomplish several missions. First, the guide should enable Texas educators to become familiar with the types of assessment strategies employed to assess learner mastery of the TEAMS mathematics objectives. For those educators wishing to become even more knowledgeable about the ways in which the TEAMS objectives are tested, the TEAMS objectives and measurement specifications, which are available at cost from the Texas Education Agency, should be consulted. For most educators, however, the level of detail provided herein will prove adequate to promote the degree of clarity needed to design on-target and effective instructional sequences. A second goal of the TEAMS Instructional Strategies Guide is to provide suggestions for instructional strategies that can be used to promote mastery of TEAMS objectives.

The specific setting in which a given district chooses to instruct the TEAMS objectives, whether in regular classes or TEAMS-focused review/remedial classes, will influence the way in which the materials in this guide will be employed. The guide is organized in a manner which will permit flexible use.

The materials contained in the following pages were developed by experienced educators, then reviewed and revised by Texas teachers and curriculum specialists.

TEAMS MATHEMATICS OBJECTIVES

MATHEMATICS: EXIT LEVEL

The TEAMS exit level mathematics objectives are measured by items that present either a numeric or word problem followed by four answer choices. Some items require students to refer to a graph, table, or geometric display to solve the problem.

The objectives tested in this part of the TEAMS exit level examination according to skill area are:

- Number Concepts
    - Number Sequence
    - Rounding
    - Equivalencies
    - Exponential Notation
- Computation
    - Fractions and Mixed Numbers
    - Decimals
    - Integers
- Problem Solving
    - Multiple Operations
    - Proportions
    - Percents
- Measurement
    - Measurement
- Geometry
    - Geometric Formulas
    - Properties of Geometric Shapes
- Probability, Statistics, and Graphing
    - Averages
    - Simple Probability
    - Graphs and Tables
- Algebraic Concepts
    - Formulas
    - Solve for a Variable

The TEAMS mathematics problems are similar to those that students might encounter in school, at home, or in their community. Because students are not permitted to use a calculator on the TEAMS test, an attempt was made to limit the size of numbers on which computations must be performed. Thus most objectives restrict numbers to four digits or less.

SKILL AREA: NUMBER CONCEPTS

The skill area of number concepts covers four objectives: number sequence, rounding, equivalencies, and exponential notation. Students will be presented with test items which require sequencing numbers from least to greatest, rounding numbers to a specified place value, identifying equivalencies between fractions, decimals, and percents, and converting numbers from exponential notation into standard notation.

Number Sequence: Select the set of numbers ordered from least to greatest.

This objective requires students to select the set of numbers that is correctly ordered from least to greatest.

Sample Item

Select the set of numbers that is correctly ordered from least to greatest.

A. -171, -135, -98, -56

B. -5, -7, 12, 2

C. 576, 289, 170, 95

D. -302, -265, -75, -101

Test Questions

1. A test item will direct the student to, "Select the set of numbers that is correctly ordered from least to greatest."

Answer Choices

1. Each answer choice will be a list of four numbers. An answer choice may combine positive, negative, integer and decimal numbers. The numbers will not be larger than four digits.

2. The correct answer choice will be the set of numbers that is correctly ordered from least to greatest.

3. An incorrect answer choice will be one of two types:

   - -- Incorrect Direction: Four numbers that are sequentially ordered from greatest to least rather than from least to greatest.

   - -- Incorrect Sequence: Four numbers that do not follow sequentially from least to greatest or from greatest to least.

Answer Analysis for Sample Item

1. Answer choice A is the correct response. The four numbers are correctly ordered from least to greatest (that is, -171 is less than -135 which is less than -98 which is less than -56).

2. Answer choice B presents numbers in an incorrect sequence. The number 12 (position 3) is larger than the number 2 (position 4). Thus, the four numbers are not correctly ordered from least to greatest.

3. Answer choice C presents numbers ordered in an incorrect direction. The four numbers are ordered from greatest to least rather than from least to greatest.

4. Answer choice D presents numbers in an incorrect sequence. The number -75 (position 3) is larger than the number -101 (position 4). Thus, the four numbers are not correctly ordered from least to greatest.

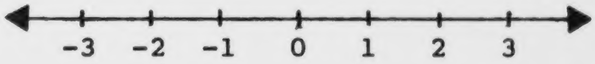
Instructional Emphases

Students must be able to identify sets of numbers ordered from least to greatest. To help students who may be having trouble with this skill you may want to review a procedure such as the following:

> To determine whether a sequence of numbers is ordered from least to greatest, begin with the number farthest to the left in the sequence. The numbers should be read from left to right. Look at each number and compare it to the number to its right. The number to the right should always be larger (greater) than the number to the left. This pattern should be consistent for the entire set of numbers. If a number does not follow in "ascending order," the response option is not correct. An example of a correct pattern is as follows: -10.5, 1.5, 12.3, 20.2. An example of an incorrect

pattern is -1.5, -10.1, 12.3, 20.2. Here, the second number in the sequence, (-10.1), is not greater than the previous number (-1.5). Thus, this pattern would be an incorrect response for a test item measuring this objective.

Consider using a number line to reinforce the following instruction:

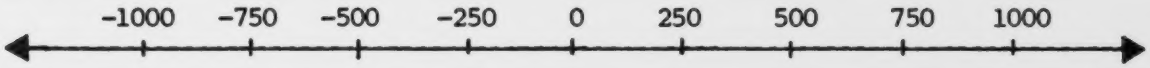


1. As you go right on the number line, the numbers are larger. As you go left on the number line, the numbers are smaller (e.g., 7.1 is less than 12.5).
2. Lower value positive numbers (such as 98) are always smaller than higher value positive numbers (such as 105).
3. Negative numbers are always smaller than zero or positive numbers (e.g., -4.2 is less than 0).
4. Negative numbers with higher absolute value are always smaller than negative numbers with lower absolute value (e.g., -105 < -98).

Possible Instructional Activities

Lining Up

Draw a number line on the board with the following numbers identified:



Then, pick a number and ask a student where the number belongs on the number line. Add the number to the number line in the appropriate place. Provide another number and again ask a student where it belongs. Be sure to include a variety of numbers (1-4 digits, positive, negative, integer, and decimal). After about 10 numbers have been added to the number line, review why each number holds its relative position.

Sizing Up States

Let students use a map of the United States and list five states in order of geographic size or population (smallest to largest) by guessing. Then have students find the actual values and order the numbers to check their guesses.

Create a Decimal Number

Divide the class into groups of four or five. Each group should be given, or should make, a set of cards that includes three of each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9--one digit per card--and six decimal point cards (of a slightly smaller size). The object of the game is to make the largest or the smallest decimal number with three digits and a decimal point. Begin by dealing one decimal point and three digit cards (facedown) to each player. Each player arranges his/her cards to form a three digit decimal number. When all players are ready, they simultaneously display their choices, announcing "least" or "greatest" as they do. The numbers are compared. The player with the smallest decimal number and the player with the largest decimal number each score one point, if they accurately described ("least" or "greatest") their number.

For variation, different numbers of digit cards can be dealt to students. In this way students must compare numbers that vary from 1 to 4 digits to determine a "winner."

Practice Time

Make a page of problems in which each problem is a list of four numbers (for example, A. -25.4, -22.9, 0, 1.56). For half of the problems, order the numbers from least to greatest. For the other half, order the numbers from least to greatest with one number out of sequence, or order the numbers from greatest to least. Have students circle the problems that are correctly ordered from least to greatest and correct the order for those that are not.

Existing Problems

Problems in mathematics textbooks on ordering integers and decimals or placing greater than (>), less than (<) symbols between numbers can provide additional practice for students.

Rounding: Round numbers to a particular place value.

This objective requires students to round a number to a specified place value ranging from thousands through thousandths.

Sample Item

Round 289.65 to the nearest whole number.

A. 300

B. 290

C. 289.7

D. 289

Test Questions

1. A test item will direct the student to, "Round (a number) to the nearest (place value)." The numbers will be whole numbers and positive decimals that are three, four, or five digits. The place values for rounding will be the thousands, hundreds, tens, ones (whole number), tenths, hundredths, and thousandths place.

Answer Choices

1. The correct answer choice will be the specified number correctly rounded to the specified place value. When rounding the number 5, the correct answer will always be rounded up. (For example: If students were asked to round 19.85 to the nearest tenth, the correct answer would be 19.9. The value 19.8 would not be presented as an answer choice.)

2. An incorrect answer choice will be one of two types:

   -- Incorrect Rounding: A number that is rounded to the correct place but is rounded incorrectly.

   -- Incorrect Place Value: A number that is rounded either correctly or incorrectly to a place value other than the one specified.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because it is rounded to an incorrect place value. The item directs the student to round to the nearest whole number, that is, to the nearest one's place. Rounding 289.65 to 300 is rounding to the nearest hundred's place, therefore, this answer choice is incorrect.

2. Answer choice B is correct because it is correctly rounded to the specified place value. The closest whole number to 289.65 is 290.

3. Answer choice C is incorrect because it is rounded to an incorrect place value. Rounding 289.65 to 289.7 is rounding to the nearest tenth, not to the nearest whole number.

4. Answer choice D is rounded to the correct place value, but involves incorrect rounding. Rather than rounding 289.65 up to the nearest whole number, it was rounded down. Because there are 6 tenths, the closest whole number is 290 not 289.

Instructional Emphases

A review of the following rounding procedures may be helpful for students. First, underline the place to which the number is to be rounded. Next, look at the number directly to the right of the underlined place. If that number is 5 or more, increase the place to be rounded by one. If that number is 4 or less, the number in the place to be rounded should remain the same. Finally, change all digits to the right of the place to be rounded to zero or drop these digits if they are to the right of the decimal point.

Present examples to students such as:

"Round 526.748 to the hundredths place."	"Round 232.55 to the tens place."
Step 1: 526.748	Step 1: 232.55
(Underline the place to be rounded)	(Underline the place to be rounded)

Step 2: $526.7\underline{4}\underset{\uparrow}{8}$

(The number 8 is greater than 5. Therefore, the place to be rounded should increase by 1.)

Step 3: $526.7\underline{5}\cancel{8}$

(Drop all decimal numbers to the right of the place to be rounded.)

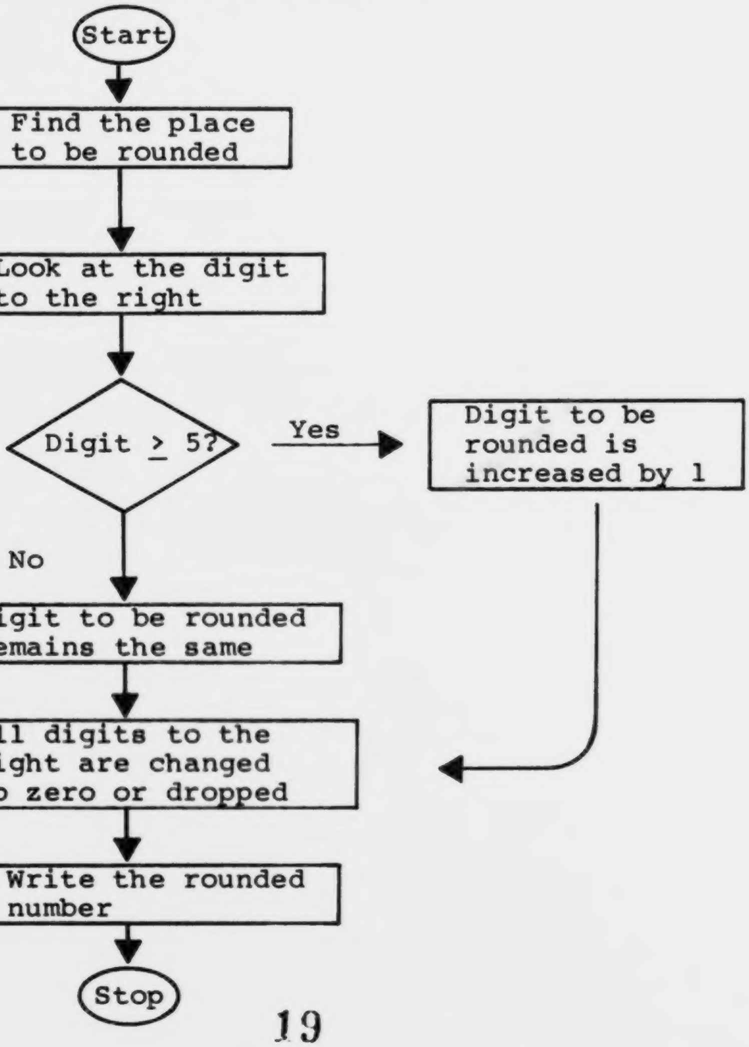
Step 2: $2\underline{3}\underset{\uparrow}{2}.55$

(The number 2 is less than 5, therefore, the place to be rounded should remain the same.)

Step 3: $2\underline{3}0\cancel{.}\cancel{5}\cancel{5}$

(Change to zero all whole numbers to the right of the place to be rounded and drop all decimal numbers and the decimal point.)

A flow chart like the following may be helpful to post while students are reviewing how to round:


Start
Find the place to be rounded
Look at the digit to the right
Digit ≥ 5?
Yes
Digit to be rounded is increased by 1
No
Digit to be rounded remains the same
All digits to the right are changed to zero or dropped
Write the rounded number
Stop

Also, be sure that students know the location of each of the following place values:

2	5	7	1	.	4	2	8
Thousands	Hundreds	Tens	Ones		tenths	hundredths	thousandths

Begin rounding instruction with whole numbers. When students have mastered this skill, introduce decimal numbers.

Possible Instructional Activities

Planet Roundup

The speed, in kilometers per second, at which each planet travels around the sun.

Mercury	47.867	Venus	35.012
Earth	29.783	Mars	24.119
Saturn	9.638	Uranus	6.806
Neptune	5.438	Pluto	4.747
		Jupiter	13.065

Have students round each number to the nearest ones, tenths, and hundredths.

A Rounded Pie

Write the decimal value of π (pi) on the board (3.14159265358973...). Explain to the class that π is the ratio of the circumference of a circle to its diameter, and that it is useful for calculating the area of circles, cylinders, or spheres. Students often find it interesting that π continues indefinitely with no repeated pattern. Ask students to round π to the ones, tenths, hundredths, and thousandths place.

Rounding Rain

Presented below are the normal annual rainfalls of 10 U.S. cities. Have students round each number to the nearest tens, ones, tenths, and hundredths.

Normal Annual Rainfall in Inches

City	Rainfall
Albuquerque, New Mexico	7.772
Amarillo, Texas	20.281
Galveston, Texas	42.203
Houston, Texas	48.194
Los Angeles, California	14.055
Mobile, Alabama	66.987
New Orleans, Louisiana	56.776
Oklahoma City, Oklahoma	31.372
Phoenix, Arizona	7.058
Salt Lake City, Utah	15.174

Number Nudging

Have students consult almanacs or other reference sources that include sets of statistical data such as the annual rainfall information presented in the previous activity. Ask students to find data sets with at least five entries, then round those numbers to each possible place value.

Sports Galore

Have students bring in current statistics from the sports page of the newspaper. Use these numbers to form rounding problems for places from the thousands to the thousandths.

Equivalencies: Identify equivalent fractions, decimals, and percents.

This objective requires students to identify equivalencies among fractions, decimals, and percents.

Sample Item

Express $\frac{4}{5}$ as a decimal.

A. .8

B. .7

C. 1.25

D. .08

Test Questions

1. A test item will require the student to carry out one of five possible conversions. These are: (1) a fraction to a decimal, (2) a fraction to a percent, (3) a decimal to a fraction in simplest form, (4) a decimal to a percent, or (5) a percent to a decimal. The test item will direct the student to, "Express (a number) as a (fraction in simplest form, decimal, or percent)."

2. Fractions will have a value of less than one and will be limited to those that can be expressed as terminating decimals with no more than three digits.

3. Decimals will have a value of less than one and will have no more than three digits.

4. Percents will be expressed using the symbol (%) and will be between .01% and 100%. Percents which contain decimals (for example, 16.25%) will be limited to a maximum of four digits with no more than two digits to the right of the decimal point.

Answer Choices

1. Each answer choice for a given test item will be either a fraction in its simplest form, a decimal, or a percent.

2. The correct answer choice will be the fraction, decimal or percent that is equivalent to the specified number.

3. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:

   - Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.
   - Incorrect Simplification: Incorrectly simplifying a fraction. This type of error will only appear as an incorrect answer choice for items directing the student to express a number as a fraction.
   - Misplaced Decimal Point: Placing the decimal point in an incorrect position on either a correct solution or an incorrect solution. Decimal errors will only appear as incorrect answer choices for items directing the student to express a number as a decimal or percent.
   - Reciprocal Error: A number that is the reciprocal of the correct answer. For example, if the correct answer is $\frac{3}{4}$, $\frac{4}{3}$ would be a reciprocal error.

Answer Analysis for Sample Item

1. Answer choice A is the correct response. The decimal equivalent of $\frac{4}{5}$ is .8 because 4 divided by 5 equals .8.
2. Answer choice B is incorrect because it contains a computation error. Four divided by 5 does not equal .7.
3. Answer choice C is incorrect because it is the reciprocal of the correct answer. Rather than dividing 4 by 5, 5 was divided by 4 yielding 1.25.
4. Answer choice D is incorrect due to a misplaced decimal point. Although the computation is accurate, the decimal is incorrectly placed. Therefore, this answer choice is incorrect.

Instructional Emphases

After stressing the importance of knowing which conversion is called for, you may want to review the following rules for each type of conversion.

Converting fractions to decimals:

1. Divide the denominator of the fraction into the numerator.

Example: $\frac{3}{4}$ = what decimal?

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{2\,8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Converting fractions to percents:

1. Convert the fraction to a decimal as shown above.
2. Multiply by 100 (move the decimal point two spaces to the right).
3. Add a percent sign.

Example: $\frac{1}{5}$ = what percent?

Step 1: $\begin{array}{r} .20 \\ 5\overline{)1.00} \\ \underline{1\,0} \end{array}$ Step 2: $.\underset{\curvearrowright}{2}\underset{\curvearrowright}{0}$ Step 3: 20%

Converting decimals to fractions:

1. Note the place value of the last digit in the decimal.
2. Drop the decimal point and place the entire number over the above-noted place value.
3. Simplify if possible.

Example: .22 = what fraction?

Step 1: $.2\underline{2}$ = hundredths Step 2: $\frac{22}{100}$ Step 3: $\frac{11}{50}$

Converting decimals to percents:

1. Multiply by 100 (move the decimal point two places to the right).
2. Add a percent sign.

Example: 0.12 = what percent?

Step 1: $.\underset{\curvearrowright}{1}\underset{\curvearrowright}{2}$ Step 2: 12%

Converting percents to decimals:

1. Drop the percent sign.
2. Divide by 100 (move the decimal point two places to the left).

Example: 17% = what decimal?

Step 1: 17% $\rightarrow$ 17   Step 2: $\underset{\curvearrowleft}{1}\underset{\curvearrowleft}{7}. = .17$

The following skills are necessary for students to master the objective of equivalencies:

1. Ability to recognize the form of fractions $\frac{A}{B}$, decimals X.XXX, and percents X.X%.

2. Ability to divide with a decimal number.

3. Ability to simplify fractions.

4. Ability to multiply and divide by 100.

If students are weak in any of these areas, they will have difficulty applying the rules for converting among fractions, decimals, and percents. Be sure to provide instruction in these areas if students are having difficulty. It may also be helpful for students to memorize a few common conversions such as .25 = 1/4 = 25%. Students can then refer to these memorized relationships to determine the method necessary to perform other less common conversions.

Possible Instructional Activities

Fill in the Blanks

Copy an equivalence chart using the following three column headings: Fraction in Simplest Form, Decimal, Percent. Use as many rows as you feel necessary. Fill in one of the spaces in each row with an appropriate number. The students are to complete the chart by converting the numbers you have supplied to their equivalencies. For example, if students are given the fraction $\frac{3}{8}$, they must convert it to an equivalent decimal and a percent in order to complete that row. At the end of the week, students should turn in their charts. You can give students credit for the number of spaces that they complete correctly.

Oral Practice

Using a mathematics textbook, review a page of fraction-decimal-percent equivalencies with students. Provide enough time for most students to solve each problem. Then, ask for volunteers to give the answer. Discuss with the entire class methods for solving the problems.

TEAMS Rehearsal

Prepare practice problems similar to the sample item for this objective. Give these problems to the class. Provide feedback regarding which answer choice is correct.

Exponential Notation: Convert numbers from exponential notation to standard notation.

This objective requires students to convert a number presented in exponential notation to an equivalent value in standard notation.

Sample Item

Simplify 6^3.

A. 18

B. 729

C. 216

D. 36

Test Questions

1. A test item will direct the student to, "Simplify (number in exponential notation)." The numbers to be converted will be whole numbers from one through ten with exponents from one through four.

Answer Choices

1. Each answer choice will be a number expressed in standard notation.
2. The correct answer choice will be the number expressed in standard notation that is equivalent to the number presented in exponential notation.
3. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:

   -- Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.

   -- Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.

   -- Incorrect Operation: Performing an operation other than the required one, specifically, adding rather than multiplying the base.

-- Incorrect Power: Multiplying the base fewer or more times than indicated by the exponent.

-- Reversal: Reversing the base with the exponent and either multiplying correctly or performing an incorrect operation or incorrect power error.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because the wrong operation was performed. The base was added rather than multiplied (6 + 6 + 6 instead of 6 x 6 x 6).
2. Answer choice B is incorrect because the base and the exponent were reversed. The value 729 is equivalent to 3^6, not 6^3.
3. Answer choice C is the correct response. The value 6^3 equals 6 x 6 x 6, which equals 216.
4. Answer choice D is the result of an incorrect power error. The value 36 equals 6 x 6, not 6 x 6 x 6. Although the correct base was used (6), it was multiplied one less time than indicated by the exponent.

Instructional Emphases

Success on this skill depends primarily on whether students have committed to memory the meaning of exponential notation. In teaching this objective, begin by distinguishing between the base and the exponent. Then, explain that the exponent indicates the number of times to use the base as a factor. Examples should help reinforce this explanation. For example, show that $2^3 = 2 \times 2 \times 2 = 8$. Also show that any whole number to the first power is always equal to the whole number, for example, $13^1 = 13$. And finally, the number 1 to any power is equal to 1, for example, $1^5 = 1$. Remember, as you develop problems, be certain to focus on bases between 1 and 10 and exponents between 1 and 4. Values other than these are not eligible for the TEAMS test.

Possible Instructional Activities

Which is Greater?

1. 2^3 or 3^2
2. 4^3 or 3^4
3. 1^5 or 5^1
4. 5^7 or 5^9
5. 1^9 or 1^2
6. 2^3 or 2×3
7. 5^2 or 5×2
8. 3^3 or 3×3
9. 9^4 or 9×4
10. 1^3 or 1×3

Present these problems to students. For each pair, ask students to circle the number with the greater value. After students finish, discuss their answers. Ask students how they solved each problem. Problems 1-3 demonstrate that different numbers are obtained if the base and the exponent are reversed. Problems 6-10 demonstrate that numbers in exponential notation do not equal the base multiplied by the exponent.

Patterns

Ask students to write each of the following as the square of a number. Discuss with students the pattern that emerges.

1. $1^3 + 2^3 = ?$
2. $1^3 + 2^3 + 3^3 = ?$
3. $1^3 + 2^3 + 3^3 + 4^3 = ?$
4. $1^3 + 2^3 + 3^3 + 4^3 + 5^3 = ?$
5. $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = ?$

Concentration

This is a good learning exercise that students can work on in pairs or small groups (perhaps when they complete an in-class assignment). Make a Concentration Game using exponents and "answers" in standard form. Write 20 numbers in exponential notation on index cards using bases from 1-10 and exponents from 1-4. On an additional 20 cards, write the corresponding number in standard notation for each exponential problem. The Concentration rules are as follows: (1) Students lay all 40 cards face down in front of them. (2) The first player turns over one card and then a second card trying to make a match between the exponential form of a number and its correct answer in standard form. (3) If a match is not made, the cards are turned face down and the next player continues in similar form. If a match is made, the player making the match collects these cards and takes another turn. That player can continue as long as a match is made each turn. (4) The game ends when all matches have been made. The winner of the game is the player who makes the most matches. Emphasize to students that this game requires concentration because they should try to remember the location of the exposed, but unmatched cards. If students can do this, they may be able to make more matches later in the game.

Create "TEAMS"

By making exponent problems using bases from 1-10 and exponents from 1-4 you can easily recreate the item pool for the TEAMS test. Use a table like the following to provide students with practice. Students should translate the number specified by the row and column to standard notation and enter it in the appropriate box.

	Exponent			
Base	1	2	3	4
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Alternately, the same table can be prepared with the numbers for the base and exponent "scrambled" rather than "in order."

Number Anagram

Present students with four numbers and a total. For example, "Use the numbers 1, 2, 3, and 4 to obtain the total, 17." Each number must be used once. In this case the solution is $1^3 + 2^4 = 17$. Problems are easy to develop by starting with the solution, e.g., $1^7 + 5^3 = 126$, then presenting the problem to students (1, 3, 5, 7 = 126).

SKILL AREA: COMPUTATION

The skill area of computation covers three objectives: fractions and mixed numbers, decimals, and integers. For each objective, students will be presented with computation problems which are described below.

Fractions and Mixed Numbers: Solve problems involving addition/subtraction/multiplication of fractions and mixed numbers.

This objective requires students to solve addition, subtraction, and multiplication problems involving mixed numbers and fractions.

Sample Item

Solve and Simplify

$$\begin{array}{r} 8 \\ -\;7\frac{5}{8} \\ \hline \end{array}$$

A. $\frac{3}{8}$

B. $\frac{5}{8}$

C. $1\frac{3}{8}$

D. $1\frac{1}{2}$

Test Questions

1. A test item will require the student to, "Solve and Simplify."
2. Numbers will be positive and will not be larger than 10. Fractions will be in simplest form with a denominator that is not larger than 32. At least one number in an item will be a mixed number or a fraction.

3. Items will be presented either vertically or horizontally. Each item will include two numbers with the exception of addition problems which may include three numbers.

Answer Choices

1. Fractions and mixed numbers will be presented in simplest form.

2. The correct answer choice will be the exact solution to the addition, subtraction, or multiplication problem presented.

3. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:

   -- Computation Error: An error in the addition, subtraction, or multiplication of two numbers.

   -- Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.

   -- Common Denominator Error: An error in selecting the lowest common denominator or in renaming fractions in terms of the lowest common denominator.

   -- Incorrect Simplification: Incorrectly simplifying a fraction.

   -- Incorrect Operation: Performing an operation other than the required one.

Answer Analysis for Sample Item

1. Answer choice A is the correct response, as illustrated below. The 8 is expressed as $7\frac{8}{8}$. Then, $\frac{5}{8}$ can be subtracted from $\frac{8}{8}$ and 7 from 7, yielding $\frac{3}{8}$.

$$\begin{array}{r} \overset{7}{\cancel{8}}\ \frac{8}{8} \\ -\ 7\ \frac{5}{8} \\ \hline \frac{3}{8} \end{array}$$

2. Answer choice B is incorrect because there is a computation error. $\frac{5}{8}$ should be subtracted from $\frac{8}{8}$, to obtain $\frac{3}{8}$ not $\frac{5}{8}$.

3. Answer choice C is incorrect because there is a regrouping error. The $\frac{5}{8}$ was correctly subtracted from $\frac{8}{8}$ yielding $\frac{3}{8}$. However, the 8 was not reduced to 7 after borrowing the $\frac{8}{8}$.

4. Answer choice D is incorrect because $\frac{3}{8}$ was incorrectly simplified to $\frac{1}{2}$.

Instructional Emphases

The problems under this skill require students to "Solve and Simplify." That means students must be familiar with the necessary operations to solve these problems and must remember to simplify their answers.

It is important for students to remember that when adding and subtracting fractions the common denominator is neither added nor subtracted, but is used as the denominator in the answer. However, in multiplication problems the denominators (whether common or not) are always multiplied together. One may choose to review multiplication of fractions before addition and subtraction because the process is somewhat less complicated.

The steps necessary to solve each type (addition, subtraction, multiplication) of problem are presented below. You may want to review these steps with students as you work through several sample problems.

Addition with Fractions and Mixed Numbers:

1. If fractions appear in 2 or 3 of the numbers to be added, check the denominators. Are they the same? If so, proceed to step 4.
2. If not, find the least common denominator (LCD). If students need help with this skill, you may want to review the following procedure using an example such as $\frac{3}{8} + \frac{1}{6}$:
    a. Find multiples of the larger number, 8 x 1 = 8, 8 x 2 = 16, 8 x 3 = 24.
    b. Divide the smaller number into the multiples until it divides evenly, $6\overline{)8}$, $6\overline{)16}$, $6\overline{)\overset{4}{24}}$.
    c. That multiple (e.g., 24) is the LCD.

3. Find equivalent fractions with the LCD.
4. Perform the addition, keeping the denominator the same.
5. Simplify the fraction, if possible.

Example 1: $5\frac{1}{3} + 2\frac{2}{3}$

Step 1: denominators are the same

Step 4:

$$\begin{array}{r} 5\frac{1}{3} \\ +\ 2\frac{2}{3} \\ \hline 7\frac{3}{3} \end{array}$$

Step 5: $\frac{3}{3} = 1 \quad 7 + 1 = 8$

Example 2: $7\frac{1}{6} + \frac{5}{12}$

Step 1: denominators are not the same

Step 2: LCD = 12

Step 3: $\frac{1}{6} = \frac{2}{12}$

Step 4:

$$\begin{array}{r} 7\frac{2}{12} \\ +\ \ \frac{5}{12} \\ \hline 7\frac{7}{12} \end{array}$$

Step 5: $\frac{7}{12}$ cannot be simplified.

Subtraction with Fractions and Mixed Numbers

Subtracting a fraction or mixed number from a whole number:

1. Convert the whole number to a mixed number having the same denominator as the fraction in the problem.
2. Perform the subtraction, keeping the denominator the same.

Subtracting a fraction or mixed number from another fraction or mixed number:

1. Check the denominators. Are they the same? If so, proceed to step 4.
2. If not, find the least common denominator (LCD).

3. Simplify the fraction, if possible.

Example: $3 - 1\frac{1}{5}$

Step 1: $3 = 2\frac{5}{5}$

Step 2:

$$\begin{array}{r} 2\frac{5}{5} \\ -\ 1\frac{1}{5} \\ \hline 1\frac{4}{5} \end{array}$$

Step 3: $\frac{4}{5}$ cannot be simplified.

3. Find equivalent fractions with the LCD.

4. Perform the subtraction, keeping the denominator the same. Remember to regroup if necessary.

5. Simplify the fraction, if possible.
Example: $9\frac{3}{8} - \frac{2}{3}$

Step 1: denominators are not the same

Step 2: LCD = 24

Step 3: $\frac{3}{8} = \frac{9}{24}$ $\frac{2}{3} = \frac{16}{24}$

Step 4:

$$\begin{array}{r} \overset{8}{\cancel{9}}\ \frac{\overset{33}{\cancel{9}}}{24} \\ -\ \ \frac{16}{24} \\ \hline 8\frac{17}{24} \end{array}$$

Step 5: $\frac{17}{24}$ cannot be simplified.

Multiplication of Fractions and Mixed Numbers

1. Convert any mixed numbers to improper fractions.
2. Cancel, if possible (optional).
3. Multiply the numerators together and the denominators together (remind students that the denominators need not be the same).
4. Simplify, if possible.

Example: $3\frac{3}{5} \times \frac{5}{9}$

Step 1: $3\frac{3}{5} = \frac{18}{5}$

Step 2: $\frac{\overset{2}{\cancel{18}}}{5} \times \frac{5}{\underset{1}{\cancel{9}}}$

Step 3: $\frac{2}{5} \times \frac{5}{1} = \frac{10}{5}$

Step 4: $\frac{10}{5} = 2$

Possible Instructional Activities

Fraction Addo

1. Have each player write the following on a piece of paper:

$$\frac{[\quad]}{[\quad]} + \frac{[\quad]}{[\quad]}$$

2. Pick 4 single digit numbers and read them aloud slowly.
3. As each number is read, students should enter it in one of the four spaces. Once a number has been written in a space, it cannot be changed.
4. Each student adds the resulting fractions. The student who has the largest sum is the winner.

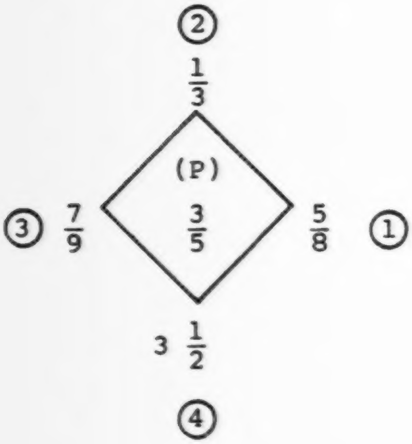
Example:

Lance	Casey	Cecilia
$\frac{1}{2} + \frac{3}{4} = \frac{5}{4} = \enclose{circle}{1\frac{1}{4}}$	$\frac{4}{3} + \frac{2}{1} = \frac{10}{3} = \enclose{circle}{3\frac{1}{3}}$	$\frac{1}{4} + \frac{2}{3} = \enclose{circle}{\frac{11}{12}}$

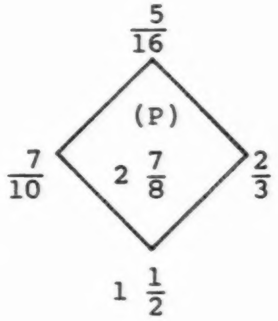
This exercise can also be used to provide practice in multiplication.

Batter Up

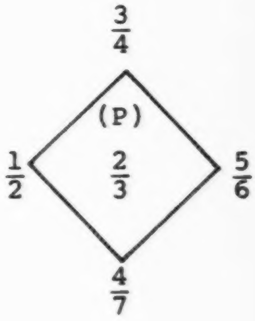
Draw a baseball diamond on the blackboard. Select a team of four students. The goal for each team is to get a home run by performing a computation: add, subtract, or multiply with a fraction/mixed number at each base and a fraction/mixed number at the pitcher's position (P). A team can earn a home run by correctly performing the specified computation at each of the four bases. (See diagram below.) Each of the four students on a team will play one base. The student at first base must add, subtract, or multiply (depending on which computation is specified in the problem) the fraction at first base with the fraction given at the pitcher's position. The second student repeats the process at second base. The third and fourth students on the team repeat the process at third base and home plate respectively. If each student on the team performs the computation correctly, the team earns a home run. This activity can be set up as a team challenge.



Add



Subtract



Multiply

Magic Squares

Magic Squares can be used to review adding and subtracting of fractions. To complete a magic square, students must first find the sum of a completely filled-in row, column, or diagonal. The sum becomes the magic sum. If the magic square is filled in correctly, each row, column, and diagonal will equal the magic sum. Using the magic sum, students should find the missing number or numbers in each row, column, or diagonal. Have students work individually to complete these magic squares.

a)

$\frac{1}{2}$	$\frac{7}{18}$	?
?	$\frac{2}{3}$	$\frac{1}{18}$
$\frac{2}{9}$	?	$\frac{5}{6}$

Magic Sum = _____

b)

?	$\frac{3}{4}$	?
?	$\frac{4}{5}$	?
$\frac{1}{2}$	$\frac{17}{20}$	$1\frac{1}{20}$

Magic Sum = _____

c)

?	$10\frac{2}{3}$	$8\frac{2}{3}$
?	?	2
$6\frac{1}{3}$	?	$11\frac{5}{6}$

Magic Sum = _____

d)

$2\frac{2}{9}$	$6\frac{17}{18}$	$7\frac{5}{6}$
$11\frac{5}{18}$	?	?
?	$4\frac{7}{18}$	?

Magic Sum = _____

Decimals: Use the basic operations to solve decimal problems.

This objective requires students to solve addition, subtraction, multiplication, and division problems involving decimal numbers.

Sample Item

$$\begin{array}{r} 40.01 \\ \underline{\text{x}\ \ 8.5} \end{array}$$

A. 322.085

B. 340.013

C. 340.085

D. 48.51

Test Questions

1. Numbers for addition and subtraction items will not be larger than four digits. For multiplication items, numbers will not be larger than a four-digit number multiplied by a two-digit number. For division items, numbers will not be larger than a four-digit number divided by a two-digit number. At least one number in an item will be a decimal number.

2. Addition and subtraction items will be presented horizontally. Multiplication items will be presented either vertically or horizontally. Division items will be presented in the form $X \div Y$ or $Y \overline{)X}$.

Answer Choices

1. The correct answer choice will be the exact solution to the addition, subtraction, multiplication, or division problem presented.

2. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:

   -- Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.

-- Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.

-- Misplaced Decimal Point: Placing the decimal point in an incorrect position on either a correct solution or an incorrect solution.

-- Misaligned Digit: (1) adding or subtracting a digit in one place to or from a digit in an adjacent place or (2) misaligning partial products.

-- Incorrect Operation: Performing an operation other than the required one.

-- Switched Operand: Reversal of the divisor and the dividend. This type of error will only appear as an incorrect answer choice for division items.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because there is a computation error, as illustrated. The first partial product should be 20005. However, a zero was accidentally dropped. This yields the error in the final product.

$$\begin{array}{r} 40.01 \\ \times\ \ 8.5 \\ \hline 2005 \\ 32008 \\ \hline 322.085 \end{array}$$

2. Answer choice B is incorrect because the digits were misaligned, as illustrated. The second partial product is not in the right position; a "place holder" zero is needed in the far right position.

$$\begin{array}{r} 40.01 \\ \times\ \ 8.5 \\ \hline 20005 \\ 320008 \\ \hline 340.013 \end{array}$$

3. Answer choice C is the correct response, as illustrated. The multiplication is accurate, the digits are correctly aligned, and the decimal is properly placed.

$$\begin{array}{r} 40.01 \\ \times\ \ 8.5 \\ \hline 20005 \\ 32008 \\ \hline 340.085 \end{array}$$

4. Answer choice D is incorrect because the wrong operation was performed. It appears that 40.01 and 8.5 were added together, rather than multiplied.

Instructional Emphases:

Because this objective calls for the use of one of four operations, it will be important for students to first identify the operation necessary to solve a particular problem. Once they have identified the operation called for, they can proceed with the necessary computations. It may help students to be reminded that computations for the decimal operations are done exactly the same as the corresponding whole number operations. You may need to spend some time, however, reviewing the placement of the decimal point.

Students should be reminded to align decimal points when setting up addition and subtraction problems. Some students may forget that the decimal point for whole numbers, which is not typically shown, follows the number.

Students should realize that it is not necessary to align decimals when multiplying or dividing. When multiplying, however, the number of places to the right of the decimal in the product is the sum of the number of places to the right of the decimal in the factors. In division, divisors should be converted from decimal to whole numbers. The decimal point in the dividend must be moved to the right the same number of places that it is moved in the divisor.

The steps required for each type of decimal operation are presented below. These steps may be helpful to review with students when you present sample problems.

Adding with Decimals

1. Align the decimal points.
2. Add, bringing the decimal point straight down from the problem into the answer.

Example: 4.2 + 2 + 3.74

Step 1:

$$\begin{array}{l} 4.2 \\ 2. \\ \underline{3.74} \end{array}$$

Step 2:

$$\begin{array}{l} 4.2 \\ 2. \\ \underline{3.74} \\ 9.94 \end{array}$$

Subtracting with Decimals

1. Align the decimal points. Add zeros following the last digit in the minuend as needed.

2. Subtract, bringing the decimal point straight down from the problem into the answer.

Example: 12.8 - 10.21

Step 1:

$$\begin{array}{r} 12.80 \\ -\ 10.21 \\ \hline \end{array}$$

Step 2:

$$\begin{array}{r} 12.\overset{7}{\cancel{8}}\overset{10}{\cancel{0}} \\ -\ 10.21 \\ \hline 2.59 \end{array}$$

Multiplying with Decimals

1. Align digits on the far right.

2. Multiply, ignoring decimals.

3. Count the number of places to the right of the decimals in the two numbers being multiplied.

4. Place the decimal point in the answer so that the number of places to the right of the decimal is the same as the number of places counted in #3 above.

Example: 7.26 (.3)

Step 1:

$$\begin{array}{r} 7.26 \\ .3 \\ \hline \end{array}$$

Step 2:

$$\begin{array}{r} 7.26 \\ .3 \\ \hline 2178 \end{array}$$

Step 3: $2.\underbrace{178}$ (3 places to the right of the decimal)

Dividing with Decimals

1. The divisor should be a whole number. Move the decimal point in the divisor to the right as necessary. Move the decimal point in the dividend to the right the same number of places. Add zeros in the dividend as needed.

2. Divide.

3. Place the decimal point in the quotient directly above the decimal point in the dividend.

Example: $.16\overline{)25.6}$

Step 1: $\underset{\curvearrowright}{1}\,\underset{\curvearrowright}{6}.\overline{)25\,\underset{\curvearrowright}{6}\,\underset{\curvearrowright}{0}.}$

Step 2:
$$\begin{array}{r} 160 \\ 16\overline{)2560.} \\ \underline{16} \\ 96 \\ \underline{96} \\ 0 \end{array}$$

Step 3:
$$\begin{array}{r} 160. \\ 16\overline{)2560\overset{\uparrow}{.}} \end{array}$$

Possible Instructional Activities

Palindromes

Tell students that numbers which read the same forward and backward are called "palindromes." The answers to each of these exercises is a palindrome.

a. $90.27 - 18$

b. $2\overline{)1191.19}$

c. $\begin{array}{r} 3.85 \\ \underline{\times\ 15} \end{array}$

d. $179.911 - 61.1$

e. $1.85 + 5 + 13.06$

f. 44.4×1.1

Write the problems on the board and have students work them out. If they work carefully and correctly, each answer will be a palindrome.

Place the Point

Give students "letter" problems and ask them to place the decimal point in the answer:

$$\begin{array}{r} \mathrm{GHI} \\ .\mathrm{AB}\overline{)\mathrm{C.DEF}} \end{array} \qquad \begin{array}{r} \mathrm{FG} \\ \mathrm{A.B}\overline{)\mathrm{CDE}} \end{array} \qquad \begin{array}{r} \mathrm{GHIJK} \\ \mathrm{AB}\overline{)\mathrm{CDE.F}} \end{array}$$

$$\begin{array}{r} \mathrm{A.BCD} \\ \underline{\times\ \ .\mathrm{EF}} \\ \mathrm{GHIJKL} \end{array} \qquad \begin{array}{r} .\mathrm{ABC} \\ \underline{\times\ \mathrm{EF}} \\ \mathrm{GHIJK} \end{array} \qquad \begin{array}{r} \mathrm{ABC.D} \\ \underline{\times\ \ .\mathrm{E}} \\ \mathrm{FGHIJ} \end{array}$$

Decimal Difficulty

Divide the class into four teams of equal abilities, then have each team create one difficult addition, subtraction, multiplication, and division decimal problem consistent with the test-item limitations for this skill (see "Test Questions" description). The 16 problems should be placed in a single practice test, then duplicated for all students. Students should take the practice test individually. The winning team is the one whose members get the most problems correct.

Practice-Practice

Because practice with decimal problems is so important, ask several students to generate practice sets of the four types of decimal problems (addition, subtraction, multiplication, and division). These students should also prepare an answer key so that other students can check on their success in completing the practice exercises. Distribute the practice exercises to the rest of the class, asking the advanced students to work on a peer-tutoring basis with students who have problems with particular types of items.

Integers: Solve problems involving addition of integers.

This objective requires students to solve addition problems involving integers.

Sample Item

(-23) + 18

A. -6

B. -5

C. 5

D. -41

Test Questions

1. The only operation tested will be addition.
2. Numbers will be integers from -100 to 100. Negative numbers will be presented in parentheses.
3. Two, three, or four numbers will be presented horizontally. At least one of the numbers will be negative.

Answer Choices

1. The correct answer choice will be the exact solution to the addition problem presented.
2. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:

    -- Computation Error: An error in the addition or subtraction of two numbers.

    -- Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.

    -- Incorrect Operation: Performing an operation other than the required one.

    -- Incorrect Sign: Using an incorrect sign on either a correct solution or a solution that is incorrect because of another error.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because there is a computation error. The error appears to be in subtracting 8 from 13 after regrouping and obtaining 6 instead of 5.

2. Answer choice B is the correct response. The problem can be solved by subtracting 23 - 18 = 5, and then assigning a negative sign because -23 is farther from 0 on the number line than is 18. The correct answer to this item, therefore, is -5.

3. Answer choice C is incorrect because it has the wrong sign. Although the arithmetic is correct, the answer should be negative, not positive. Choice C is, therefore, incorrect.

4. Answer choice D is incorrect because a positive 23 was added (23 + 18) rather than a negative 23. A negative sign was then placed on the sum.

Instructional Emphases

Students are required to solve problems involving addition of integers that include at least one negative number. Explain to students that adding a negative number is the same as subtracting a positive number. You may want to encourage students to use a step-by-step approach such as the following:

1. Add all the positive numbers together.

2. Add all the negative numbers together.

3. Compare the absolute values of the numbers obtained in Steps 1 and 2 above. Subtract the number with the smaller absolute value from the number with the larger absolute value.

4. The answer should be given the sign (+, -) of the number with the larger absolute value.

Example 1: (-21) + 18 + 20

Step 1:

$$\begin{array}{r} 20 \\ +\ 18 \\ \hline 38 \end{array}$$

Step 2: (-21)

Step 3: $\begin{array}{r} 38 \\ -\ 21 \\ \hline 17 \end{array}$

Step 4: 17 (The sign of 38 is positive)

Example 2: 6 + (-32) + 12 + (-5)

Step 1: $\begin{array}{r} 12 \\ +\ \ 6 \\ \hline 18 \end{array}$

Step 2: $\begin{array}{r} (-32) \\ +(-\ 5) \\ \hline -37 \end{array}$

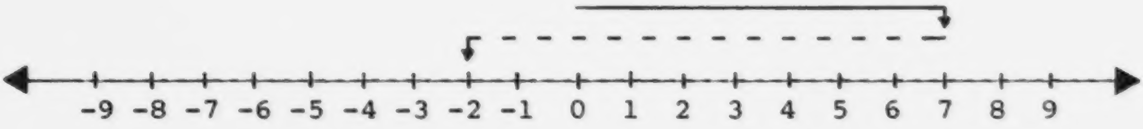
Step 3: $\begin{array}{r} 2\,17 \\ \cancel{3}\cancel{7} \\ -\ 18 \\ \hline 19 \end{array}$

Step 4: -19 (The sign of 37 is negative)

Possible Instructional Activities

On the Line

Have students practice adding integers by using a number line. Draw a number line on the board. Review with students how to solve a problem such as 7 + (-9): Start at zero and move 7 units in the positive direction. From there, move 9 units in the negative direction. The number at which you stop is the answer to the problem. (In this case, -2.)



Ask for volunteers to go to the board and illustrate the solutions of problems using a number line. Give each student a different problem.

Math Maze

Draw this diagram on the board:

		7	Start
-3	4	-2	
5	2	-3	
7	-1	-8	
	Finish (final sum = 8)		

Ask a student to volunteer to trace a path through the diagram from Start to Finish. The path must use every number once, but not reuse any numbers. Tell the student to move from one square to the next, adding the integers as he/she proceeds. Movement can only be horizontal or vertical. The number opposite "Finish" is the final sum. Have other students try these:

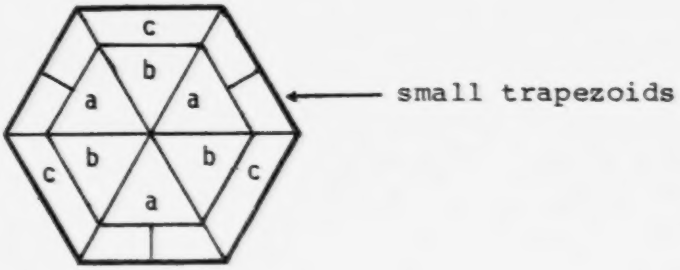
		-6	Start
8	-4	3	
2	5	7	
-3	-4	-6	
			Finish (2)

	4		Start
-1	-3		
7	-2	-5	
-4	-3	8	
			Finish (1)

If desired, have students make up their own integer diagrams for extra credit, or for an in-class activity.

Hexa-Plex

Help students create their own Hexa-Plex like the one shown below (without the letters).



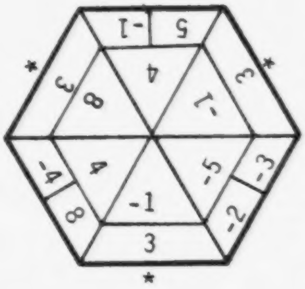
Let the students choose positive and negative integers to put in the small trapezoids in the outer spaces. (See above diagram for placement.) Then have students follow these instructions:

a. Add the pairs of digits in the small trapezoids and put the sums in the small triangles directly above the small trapezoids. (See "a" on the diagram above.)

b. Add each pair of the sums (in the small triangles) to get a new sum and place it in the triangle between them. (See "b" on the diagram above.)

c. Find the sums of the pairs of numbers in opposite triangles and put the answers in the remaining outer trapezoids. (See "c" on the diagram above.)

The last three sums will always be the same. Here's an example:



Have students substitute other integers into the hexa-plex following the same three steps outlined above to see whether they can find a combination of numbers that does not work, that is, a combination that does not yield the same three numbers after step c.

More Magic Squares

Have students create a magic square using these integers: -4, -2, -1, 0, 1, 2, 3, 4, 6. The magic sum for this set of numbers is 3. Here's the magic square format:

To complete the magic square, students should place the given integers in the squares (one integer per square). Each integer may only be used once. The trick is to place the numbers so that all rows, columns, and diagonals add up to the magic sum (which, in this case, is 3).

SKILL AREA: PROBLEM SOLVING

The skill area of problem solving includes three objectives: multiple operations, proportions, and percents. Students will be presented with word problems covering each of these objectives. Some of the problems will be based on content from the area of personal finance.

Multiple Operations: Solve word problems involving multiple operations.

This objective requires the student to solve word problems involving multiple operations on whole numbers, mixed numbers, fractions, and decimals.

Sample Item

José works at a car rental business. The fee for car rental is $20 per day plus $.15 per mile. A customer rented a car for 6 days and drove 500 miles. How much should José charge the customer for the car rental and mileage?

A. $120

B. $150

C. $195

D. $ 45

Test Questions

1. A test item will present a word problem whose solution requires two or three operations. The operations tested will be addition, subtraction, multiplication, and division.

2. Numbers will be whole numbers, mixed numbers, fractions, and decimals. They will be positive and will not be larger than four digits. Fractions will have a numerator with a value from 1 through 9 and a denominator with a value from 2 through 10. If a test item contains both decimals and fractions, the numerator of the fraction will equal 1.

3. All operations will result in positive numbers.

4. Addition operations will require the student to add two or three numbers. All other operations will involve only two numbers.

5. For multiplication operations, numbers will not be larger than a four-digit number multiplied by a two-digit number.

6. For division operations, numbers will not be larger than a four-digit number divided by a two-digit number. The division of fractions and mixed numbers will not be required.

7. Some test items will contain numerical information that is irrelevant to the solution of the word problem presented.

Answer Choices

1. The correct answer choice will be the exact solution to the problem presented.

2. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:

    -- Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.

    -- Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.

    -- Misplaced Decimal Point: Placing the decimal point in an incorrect position on either a correct solution or an incorrect solution.

    -- Misaligned Digit: Either (1) adding or subtracting a digit in one place to or from a digit in an adjacent place or (2) misaligning partial products.

    -- Incorrect Order: Performing the operations required to solve the problem in the wrong order.

    -- Missing Operation: Not operating on the data or omitting one of the required operations.

    -- Incorrect Data Error: Performing a necessary operation on the wrong data.

    -- Incorrect Operation: Performing an operation other than a required one.

-- Common Denominator Error: An error in selecting the lowest common denominator or in renaming fractions in terms of the lowest common denominator.

-- Incorrect Simplification: Incorrectly simplifying a fraction.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because of a missing operation. There are two different car rental charges presented in the test item: $20 per day and $.15 per mile. This answer choice equals the per day rental charge multiplied by the number of days rented ($20 x 6 = $120). It omits the cost per mile.

2. Answer choice B is incorrect because the digits are misaligned in the multiplication of the price per mile and the number of miles driven, as illustrated below:

```
Step 1: (no error)  $20 per day
                    x  6 days
                    $120

Step 2: (error: misaligned        500 miles
         digit in second         x .15 per mile
         partial product)         2500
                                  500
                                $30.00

Step 3: (incorporates             $120
         error from Step 2)      +  30
                                  $150
```

3. Answer choice C is the correct response, as illustrated below:

```
Step 1:   $120 per day
          x  6 days
          $120

Step 2:    500 miles
         x .15 per mile
          2500
          500
        $75.00

Step 3:   $120
         +  75
          $195
```

4. Answer choice D is incorrect because the wrong operation is performed in Step 3, as illustrated below:

Step 3: (error: subtract rather than add numbers from Steps 1 & 2)

$$\begin{array}{r} \$120 \\ -\ 75 \\ \hline \$\ 45 \end{array}$$

Instructional Emphases

For this objective, students sometimes have more difficulty determining the appropriate operations to perform than actually performing the operations. Therefore, instruction and practice on translating word problems into mathematical operations is essential. Because not all data presented in a word problem are necessary, students must be able to distinguish between essential and irrelevant information. You may want to review a step-by-step procedure such as the one outlined below.

How to Solve Word Problems Involving Multiple Operations

1. Read the problem.

   Read the problem once to get the general idea of what the problem is about. Pay particular attention to the question being asked.

2. Find the essential information.

   Carefully reread each piece of information provided in the problem. Determine which pieces of information are relevant to answer the question. Make notes on scratch paper about the key information.

3. Determine the computations to be performed and their order.

   Based on the question, determine the necessary computations. Sometimes a simple equation will be helpful to describe the operations to be performed.

4. Perform the necessary computation(s).

   After each computation is performed, refer to the word problem to check that you are proceeding correctly. Also, double-check the accuracy of all computations.

5. Is the answer reasonable?

Check that your answer makes sense based on the information presented in the problem. Incorrect operations or computation errors will sometimes result in answers that are too small or too large to be reasonable within the context of the problem.

The sample item can be used to illustrate each of the steps described above:

Step 1. The problem is about car rental costs. The question asks about the charge for a particular customer.

Step 2. The important information includes:

a. $20 per day for 6 days of rental
b. $.15 per mile for 500 miles

All numerical data presented in this problem are essential.

Step 3. To find the total cost, find the cost for 2a and 2b above, then add:

$$20(6) + .15(500) = X$$

Step 4.

$$\begin{array}{r} 20 \\ \times\ 6 \\ \hline 120. \end{array} \qquad \begin{array}{r} 500 \\ .15 \\ \hline 2500 \\ 500 \\ \hline 75.00 \end{array} \qquad \begin{array}{r} 120 \\ +\ 75 \\ \hline \$195 \end{array}$$

Step 5. The answer appears reasonable given the costs described in the problem.

Possible Instructional Activities

Mail Order

Bring in (or have students bring in) mail-order catalogs or newspaper advertisements which list several items to be sold and the corresponding prices. Have students write word problems using the materials. For example:

> Shirts are on sale 2 for $28.00. If Lee buys 3 shirts, how much will he spend?

Direct students to read their problems aloud for the class to solve. Have students express each problem in terms of an equation, if possible. Discuss how each problem can be solved.

Comparison Shopping

Give students the following problems for practice on multiple operations:

Compare the prices of each pair of different size products below. Which size is the better buy? How much do you save per ounce?

	Size	Price	Size	Price
Liquid detergent	32 oz.	$2.40	64 oz.	$4.48
Orange juice	12 oz.	$1.32	16 oz.	$1.44
Tuna	6 oz.	$ .69	8 oz.	$1.08
Soda	6(12 oz) cans	$2.16	64 oz.	$1.28

Working-Wise

Problems about salaries or wages can be developed that require multiple operations. Students may be able to help provide ideas from their experience. When possible, include extraneous information as in problem 2 below.

1. A print shop pays part-time help $5.50 an hour and 1-1/2 time for overtime hours (after 5:00 p.m. and on weekends). If an employee worked 20 regular hours and 6 overtime hours, how much should the employee earn?

2. Each week that employees work at Hayley Electronics, they earn .4 of a day vacation time. If an employee worked for 6 months (24 weeks) at $7.00 per hour and took 2 vacation days, how many vacation days would the employee have remaining?

Pizzeria

Obtain several take-out menus from local pizza restaurants. Use the menus to construct word problems involving different size pizzas and different numbers of ingredients. Include extraneous information to provide students with practice in distinguishing essential from irrelevant information.

Proportions: Solve word problems involving proportions.

This objective requires students to solve word problems involving proportions. Students will be asked to solve for a missing quantity.

Sample Item

You are in charge of getting bread for the senior picnic. You were told that 3 loaves of bread would serve 22 people. If there are 330 people going to the picnic, how many loaves of bread will you need?

A. 45 loaves

B. 66 loaves

C. 35 loaves

D. 15 loaves

Test Question

1. A test item will present a word problem that involves a proportion, that is, the application of a ratio to a quantity. The ratio will be in simplified form with a numerator from 1 through 9 and a denominator from 1 through 99. The given quantity will be a whole number that will not be larger than 3 digits.

Answer Choices

1. Each answer choice will be a whole number labeled with the appropriate unit from the word problem.

2. The correct answer choice will be the exact solution to the problem presented.

3. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:

   -- Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.

   -- Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.

   -- Missing Operation: Not operating on the data or omitting one of the required operations.

-- Incorrect Operation: Performing an operation other than a required one.

-- Incorrect Data Error: Performing a necessary operation on the wrong side.

Answer Analysis for Sample Item

1. Answer choice A is the correct response, as illustrated below. The proportion is set up correctly and all computations are accurate:

Step 1: (set up) $\frac{3}{22} = \frac{X}{330}$

Step 2: (cross product) $3 \times 330 = 22X$

Step 3: (multiplication) $990 = 22X$

Step 4: (set X alone) $\frac{990}{22} = X$

Step 5: (division)

$$\begin{array}{r} 45 \\ 22\overline{)990} \\ \underline{88} \\ 110 \\ \underline{110} \end{array}$$

Answer: $45 = X$

2. Answer choice B is incorrect because the wrong operation was performed. Rather than dividing 3 by 22, the two numbers were multiplied together, yielding 66. Students who select this choice probably did not set up a proportion.

3. Answer choice C is incorrect because it contains a regrouping error. Students who select choice C are likely to have performed operations on the data without formally setting up a proportion:

$$\begin{array}{r} 15 \\ 22\overline{)330} \\ \underline{22} \\ 110 \\ \underline{110} \end{array} \qquad \begin{array}{r} 15 \\ \underline{\times \;\; 3} \\ 35 \end{array} \text{ (error)}$$

4. Answer choice D is incorrect due to a missing operation. Students who select this answer choice probably approached the problem as in #3 above, but dropped the second step. The value of "3" from the word problem was never included in the calculations.

Instructional Emphases

Students should be encouraged to solve proportion problems by forming an equality between two ratios and solving for the missing value ($\frac{A}{B} = \frac{C}{D}$). There are numerous key words that students can look for to help them recognize the ratio that is presented. For example, the word "to" is often used, such as wins to losses or teachers to students. Other words that indicate a ratio include "for" (2 gallons of ice cream for every 6 people), "in" (6 hours of sleep in 24 hours), "out of" (18 problems right out of 21 problems), or "on" (100 miles on 4 gallons of gas). Once students identify the ratio they should express it in the form $\frac{A}{B}$, for example,

$$\frac{4 \text{ gallons}}{100 \text{ miles}}$$

Every proportion problem will provide a third related value such as the number of miles to be traveled (for example, 500 miles). Students should enter this value in the corresponding upper or lower half of the equality. For example:

$$\frac{4 \text{ gallons}}{100 \text{ miles}} = \frac{C}{500 \text{ miles}}$$

Because the additional value presented was miles, it was entered in the lower half of the equality. If the number of gallons to be purchased (for example, 20) were given, it would have appeared in position C:

$$\frac{4 \text{ gallons}}{100 \text{ miles}} = \frac{20 \text{ gallons}}{D}$$

Once the proportion is specified, students will need to cross-multiply and solve for the missing value:

$$4(D) = 100(20) \qquad 4D = 2000 \qquad D = \frac{2000}{4} \qquad D = 500$$

Let's review these procedures in specific steps that students can use as guidelines to check their work.

Step 1: Identify the ratio specified in a proportion problem and express it as:

$$\frac{A}{B}$$

Step 2: Identify the additional value specified in the problem.

If it is in the same units as value A, place it in the upper half of the equality:

$$\frac{A}{B} = \frac{C}{?}$$

If it is in the same units as value B, place it in the lower half of the equality:

$$\frac{A}{B} = \frac{?}{D}$$

Step 3: Cross multiply and divide.

$$A(?) = BC \qquad \text{or} \qquad AD = B(?)$$

$$? = \frac{BC}{A} \qquad\qquad \frac{AD}{B} = ?$$

Possible Instructional Activities

Milk Anyone?

Write the following recipes on the chalkboard.

Creamy Caramel-flavored Milk	Monkey Juice
8 tablespoons of caramel syrup	2 bananas, mashed
4 glasses of milk (1 quart)	4 glasses of milk (1 quart)

Ask students to identify the ratio within each recipe. Then have students write a proportion equation to determine how many tablespoons of caramel syrup or how many bananas are needed for:

		Caramel	Banana
a.	2 glasses of milk?	______	______
b.	6 glasses of milk?	______	______
c.	16 glasses of milk?	______	______

Ask for volunteers to write the proportion equations (one each) on the board and explain how they set up the equations. Have students solve each equation.

This activity can be repeated using more complex recipes. You may want to have students make up their own proportion problems using recipes brought from home. Remember, fractional answers will not be tested for this skill on the TEAMS test.

Take a Poll

Take a poll in class of students' plans for the future or their favorite sports. Then, using that data, have students make predictions about students in the school. For example, if 22 out of 30 students in the class plan to go to college, predict how many students in the school plan to go to college. (Students will need to know the number of students in the school for this activity.)

As homework, have students take a poll of approximately 20 people. Some ideas for a poll include: (1) How many people have a dog, a cat, or a rabbit for a pet? or (2) How many people prefer apples, peaches, or oranges? Have students record their information like this: Out of a sample of 20 people, 8 people liked apples best, 9 people liked peaches best, and 3 people liked oranges best. Then, use the students' information to discuss predictions. Have students write a prediction question below their data such as, out of 400 people, how many people would you predict would prefer apples, peaches, or oranges as their favorite fruit? Have students exchange their papers and solve the prediction problems.

Party Planning

Tell students to imagine that they are planning a party for 40 guests. Friends have given them advice about how much food and drink to buy. Ask students to read the "advice" and use it to determine the quantity of each item to buy for "their" party.

Hamburger: 1 lb for 4 people
Hot dogs: 3 hot dogs for 5 people
Potato chips: 5 ounces for 2 people
Soft drinks: 12 cans for 5 people
Ice cream: 1 carton for 5 people

To develop additional practice exercises, think of other types of activities that require planning. For example, ask students to determine the amount of gas, or time needed for a trip based on shorter distances.

Percent: Solve word problems involving percents.

This objective requires the student to solve word problems involving percents. These will be of two types: (1) using a percent to determine a quantity; or (2) using a quantity to determine a percent.

Sample Item

After 28 games, the Texas Blockers had won 75% of their games. How many games had they won?

A. 7 games

B. 20 games

C. 2 games

D. 21 games

Test Questions

1. A test item will present either:
   a. a percent and ask for the quantity that the percent represents (30% of 120 = [quantity]), or
   b. a number that is part of a total quantity and ask for the percent that the number represents (36 out of 120 = [percent]).
2. For items that require the student to determine a percent, the total quantity will either be presented in the word problem or can be computed by a sum of the parts.
3. Numbers will be whole numbers and positive decimals not larger than four digits.
4. Percents will range from 0% to 100%.

Answer Choices

1. The correct answer choice will be the exact solution to the problem presented.
2. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:

-- Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.

-- Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.

-- Missing Operation: Not operating on the data or omitting one of the required operations.

-- Incorrect Operation: Performing an operation other than a required one.

-- Incorrect Data Error: Performing a necessary operation on the wrong data.

-- Misplaced Decimal Point: Placing the decimal point in an incorrect position on either a correct solution or an incorrect solution.

-- Reciprocal Error: A number that is the reciprocal of the correct answer.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because incorrect data are used. Rather than multiplying by .75, 28 is multiplied by 25% (.25).

2. Answer choice B is incorrect because there is a regrouping error, as illustrated below. The second partial product is inaccurate. This yields the error in the final product.

   Step 1: (change percent to decimal; no error) 75% = .75

   Step 2: (error)

$$\begin{array}{r} .75 \\ \times\ \ 28 \\ \hline 600 \\ 140\ \ \\ \hline 20.00 \end{array}$$

3. Answer choice C is incorrect due to a misplaced decimal point. The decimal point is incorrectly placed on the error described in #2 above.

4. Answer choice D is the correct response, as illustrated below. The problem is set up correctly, and all computations are accurate.

Step 1: (change percent to decimal) 75% = .75

Step 2: (perform multiplication)

$$\begin{array}{r} .75 \\ \times\ \ 28 \\ \hline 600 \\ 150\ \ \\ \hline 21.00 \end{array}$$

Instructional Emphases

Because this objective requires students to do one of two types of problems, the first step is to identify which problem type is involved. The last sentence of each problem provides that information. It will either ask what percent..., or how much... or how many ... For the "what percent" items, students will have to determine a percent from a total quantity. The "how much" or "how many" items require students to determine a quantity that a specified percent represents. Once students establish the type of problem they must solve, they should then focus on isolating the information needed to solve the problem.

To review, students should identify which problem type is involved [to determine a quantity (how much or how many) or to determine a percent (what percent)]. If the problem asks for a percent, students must determine the total quantity (it's either given or can be computed). The next step in solving these problems is to perform the appropriate computations. You may want to review the computations outlined below with your students.

Computing Percent Problems

A. Determining a quantity ("how many" or "how much" that a specified percent represents (For example, 25% of 120 = ? quantity)

1. Translate the percent into a decimal by dividing by 100 (move the decimal point to the left two places: $.\underset{\curvearrowleft}{2}\underset{\curvearrowleft}{5} = .25$).

2. Multiply the decimal times the total quantity: .25 (120) = 30.

3. Check that the answer is reasonable given the percent and number specified.

B. Determining a percent from a given quantity (For example, 6 out of 8 = ? percent)

1. To find the percent, form a ratio of the number of interest (6) to the total (8):

$$\frac{6}{8}$$

2. Perform the division indicated by the ratio:

```
   .75
8)6.00
  5 6
   40
   40
    0
```

3. Multiply by 100 (move the decimal point to the right two places) to obtain the percent, 75%.

$$.\underset{\curvearrowright}{7}\underset{\curvearrowright}{5} = 75\%$$

4. Check that the answer is reasonable given the relative size of the numbers in the ratio.

Alternately, you may want to teach percent problems through the use of proportions. This way all percent problems can be solved in the same way:

$$\frac{\text{rate}}{100} = \frac{\text{part}}{\text{whole}}$$

Students must first be able to label each part of a percent problem. For example, 25% of 200 is 50.
rate whole part
Thus, if a problem gives a percent and a whole, e.g., 25% of 120 = ?, the proportion is completed as follows:

$$\frac{25}{100} = \frac{N}{120}$$

Students should then cross multiply and solve for N:

$$120 \times 25 = 100N$$
$$3000 = 100N$$
$$N = 30$$

If a problem presents a part and a whole, e.g., 6 out of 8 = ?%, then the proportion is completed as follows:

$$\frac{N}{100} = \frac{6}{8}$$

$$8N = 6 \times 100$$
$$8N = 600$$
$$N = \frac{600}{8}$$
$$N = 75$$

Students should be aware that for some percent items, the total quantity will not be provided outright in the word problem. In such cases, students can compute the total quantity by summing the parts. Thus, students must determine whether the total quantity is provided or whether it must be computed before solving this type of percent problem.

Possible Instructional Activities

Blow-Out Sale

Make signs which advertise various sale items or find advertisements in newspapers or magazines. Have students compute the amount of savings or the percent of savings for each item. Be sure to explain, or have a student explain, how the correct answer is computed.

Sign Ideas:

Find the amount of savings.

T-Shirts
Regularly $10.99
Now 20% off

$2.20 savings

Tennis Balls
10% off
our regular price
of $2.50

$0.25 savings

Album Sale
Regularly $8.98
Now 15% off

$1.35 savings

Find the percent of savings.

Shorts
Formerly $12.00
Save $3.00

25% off

Today's Special
Donuts
Regularly $3.50/dozen
Save $1.75/dozen

50% off

Hats
Were $8.50
Save $1.70

20% off

Classroom Characteristics

These problems are written so that students can get the information they need for the activity simply by observing their classmates. This will keep the noise level at a minimum. You can, however, add such things as birthdays during a certain month, eye color, favorite foods, or favorite music groups. If you decide to add these types of questions, you may wish to devise a system for students to obtain the information they need to solve their problems without disrupting the class. One way would be to use a show of hands in response to each question. With this approach, either you or a student could read the questions and the other students could tally the information that they need.

1. About 40% of the population in the U.S. has brown hair. Based on this percentage how many students in this class would you expect to have brown hair? What is the actual percent of students in this class with brown hair?
2. What percent of the students in this class are wearing yellow (or any other color)?
3. About 52% of the population in the U.S. are females. Based on this percentage how many students in this class would you expect to be female? What is the actual percent of students in this class who are female?
4. What percent of the students in this class are wearing a watch?

Note: When using this activity, the "total quantity" portion of the problems will always be the number of students in the class. You may want to stipulate that the problems should be solved using the students in the class at the time of the activity. This will eliminate the confusion that may arise when students try to account for absent classmates.

Chocolate Flakes

Explain to students that nutritionists are concerned with the amount of vitamins, minerals, and other nutrients in foods. Food processing companies often hire nutritionists to determine nutrition information for their products. That information usually appears on the packages that contain the products. For example, the side panel of a box of cereal lists the nutrients found in one serving of the cereal. One serving of Chocolate Flakes contains about 1,250 International

Units of vitamin A. The recommended daily allowance of vitamin A is 5000 International Units (I.U.). What percent of the recommended daily allowance of vitamin A is contained in one serving of Chocolate Flakes?

For each nutrient listed below, compute the percent of the recommended daily allowance that is contained in one serving of Chocolate Flakes.

Nutrient	Recommended Daily Allowance	Amount in One Serving of Chocolate Flakes
Vitamin D	400 I.U.	40 I.U.
Vitamin C	45 mg	11.25 mg
Thiamine	1.3 mg	.325 mg
Riboflavin	1.6 mg	.4 mg
Niacin	16 mg	4 mg
Iron	18 mg	1.8 mg
Protein	50 g	1 g
Phosphorus	800 mg	16 mg

Percentages of the U.S.

Have students examine a map or do research on the United States in order to answer historical and geographical questions involving percent. For example, "What percent of the 50 states have a mandatory seat belt law?" "What percent of the 50 states have a common border with Mexico?" "What percent of the 50 states begin with the letter A?"

You may want to assign questions or allow students to choose their own. If you desire, you can have students orally present their answers to the class.

Tilt the Scale

Explain to students that about 18 percent of their total body weight is bone, about 50 percent is muscle, and about 20 percent is fat tissue. Have students use these percents to find the approximate weight of bones, muscle tissue, and fat tissue for someone who weighs 150 pounds.

A second, and related activity is as follows: A total of 22 percent of the weight of the bones is water, 76 percent of the muscles are made up of water, and 20 percent of the fat tissue is water. Have students find the approximate weight of the water in the bones, muscle tissue, and fat tissue combined. Again, have students solve this problem for someone who weighs 150 pounds.

Answers

Problem 1:	27 lbs. bone	Problem 2:	5.94 lbs. water in bone
	75 lbs. muscle		57 lbs. water in muscle
	30 lbs. fats		6 lbs. water in fat
			68.94 lbs.

Daily Activities

Have students keep track for one day the approximate amount of time (to the nearest hour) they spend on the following activities. The time must total 24 hours.

Activity	Time
Sleeping	____
Eating	____
Classes	____
Studying	____
Watching TV	____
Talking on Phone	____
Socializing with friends	____
Working	____
Others:	
____________________	____
____________________	____
____________________	____

After students have monitored their activities for a day, ask them to find the percent of time they spend on each activity. Some activities can be combined and new percents determined, for example, the percent of time spent in classes or studying.

SKILL AREA: MEASUREMENT

Measurement: Solve problems involving metric/customary measurements using basic operations.

For this skill students will be required to solve word problems involving measurement data presented in either metric or customary units.

Sample Item

Maureen has a new job at Hungry Burger. The restaurant is famous for its milk shakes. She was told to use 3 ounces of milk for each shake. After practice, the baseball team went to the Hungry Burger and ordered 22 milk shakes. How much milk did Maureen need for those milk shakes?

A. 1 quart 24 ounces

B. 2 quarts 2 ounces

C. 1 quart 30 ounces

D. 3 quarts 3 ounces

Test Questions

1. A test item will present a word problem that requires the use of one operation (addition, subtraction, multiplication, or division) on either metric or customary measurements.

2. A test item will require a conversion between units of measure within one system of measurement.

3. A test item will not include both metric and customary units and will not require conversions between the two measurement systems.

4. Numbers for use with metric units will be whole numbers and positive decimals and will not be larger than four digits.

5. Numbers for use with customary units will be whole numbers, fractions, and mixed numbers. All fractions will have a numerator from 1 through 9 and a denominator from 1 through 10. Mixed numbers will not have a value larger than 1,000.

6. Eligible units of measure will be:

METRIC	CUSTOMARY
Length	
kilometer	mile
meter	yard
centimeter	foot
millimeter	inch
Area	
length units squared	length units squared
Volume and Capacity	
liter	gallon
milliliter	quart
	pint
	cup
	ounce
Weight and Mass	
kilogram	pound
gram	ounce
milligram	

Time

year
month
week
day
hour
minute
second

7. A table of conversions for each system, except for time, will be provided in the test booklet.

Answer Choices

1. The correct answer choice will be the exact solution to the problem presented.
2. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:

-- Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.

-- Conversion Error: Using an incorrect conversion constant.

-- Conversion Regrouping Error: Using a correct conversion constant but regrouping between units incorrectly.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because there is a computation error in Step 2, as illustrated below.

   Step 1: (total number of ounces; no error) 22 (3 oz.) = 66 oz.

   Step 2: (conversion to quarts; subtraction error)

$$\begin{array}{r} \text{1 quart 24 ounces} \\ 32\overline{)66} \\ \underline{32} \\ 24 \end{array}$$

2. Answer choice B is the correct response. The problem is set up correctly, the computations are accurate, and the unit conversion is correct.

   Step 1: (total number of ounces) (22) (3 oz.) = 66 oz.

   Step 2: (conversion to quarts)

$$\begin{array}{r} \text{2 quarts 2 ounces} \\ 32\overline{)66} \\ \underline{64} \\ 2 \end{array}$$

3. Answer choice C is incorrect because there is a conversion error. Using the incorrect conversion constant 1 quart = 36 ounces instead of 1 quart = 32 ounces yields the answer, 1 quart 30 ounces. This error is shown in Step 2, below.

   Step 2: (conversion to quarts; conversion constant error)

$$\begin{array}{r} \text{1 quart 30 ounces} \\ 36\overline{)66} \\ \underline{36} \\ 30 \end{array}$$

4. Answer choice D is incorrect because there is a multiplication error in Step 1, 22(3oz.) = 99 oz.

Instructional Emphases

Measurement problems involve two distinct steps. Students must perform one of four basic operations (addition, subtraction, multiplication, or division) and students must perform a conversion to simplify their answer. First, students should determine the appropriate operation and perform that operation on the values presented. If the values are presented in two units, e.g., 2 gallons and 1 quart, students should perform the operation on each, keeping the units separate. Second, students must simplify to obtain the correct answer. For example, if a problem required 2 gallons and 1 quart to be multiplied by 4, the answer, 8 gallons and 4 quarts, must be simplified to 9 gallons.

You may want to familiarize students with conversion charts for both metric and customary units. Students will be provided with conversion charts for each system, except for time. Thus, memorization of all conversions is unnecessary. It is, however, essential for students to familiarize themselves with the charts and to demonstrate the ability to use them correctly.

Possible Instructional Activities

Convert It

Using a conversion chart, complete the conversions below.

5 cups = ____ pint ____ cup	(2 pt 1 cup)
6482 grams = ____ kilograms	(6.482 kg)
7 pints = ____ quarts ____ pints	(3 qt 1 pt)
2398 milliliters = ____ liters	(2.398 l)
193 centimeters = ____ meters	(1.93 m)
6.72 grams = ____ milligrams	(6720 mg)
16 quarts = ____ gallons	(4 g)
30 ounces = ____ pounds ____ ounces	(1 lb 14 oz)
30 millimeters = ____ centimeters	(3 cm)
15 months = ____ years ____ months	(1 yr 3 mos)

Create other conversions for additional practice.

Ounce for Ounce

Have students keep track of how many ounces of soda they drink each day for a school week (five days). Students should then total the number of ounces and simplify the total to quarts or gallons. To adapt this activity, substitute another liquid for soda or use another food item. When choosing items for this activity, be sure that the weight or volume is easily determined.

Ruler Happy

Have students measure their heights in inches or centimeters. Then, have students convert their height from inches to feet and inches or from centimeters to meters. Students may add their "neighbors" height to their own, then, simplify that value. Objects around the classroom can also be measured and then converted into different units of measure.

SKILL AREA: GEOMETRY

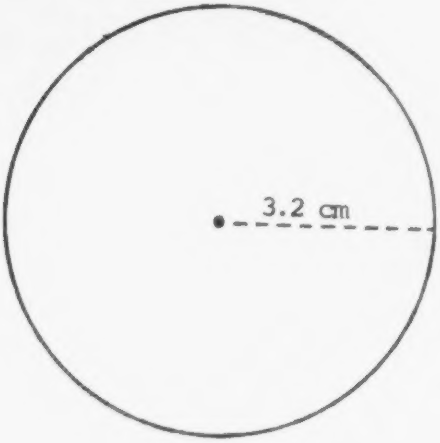
The skill area of geometry covers two objectives: geometric formulas and the properties of geometric shapes. Geometric formulas require students to compute the area, perimeter, circumference or volume of a geometric figure. Properties of geometric shapes require students to find the length of a missing side or size of a missing angle of a geometric figure.

Geometric Formulas: Solve problems involving geometric formulas.

This objective requires the student to solve problems involving geometric formulas.

Sample Item

Find the area of the circle below.



A. 6.4 πcm^2 (20.096 cm^2)

B. 3.2 πcm^2 (10.048 cm^2)

C. 9.24 πcm^2 (29.0136 cm^2)

D. 10.24 πcm^2 (32.1536 cm^2)

Test Questions

1. A test item will either:

   a. present a labeled display and direct the student to, "Find the (area, perimeter, circumference, volume) of the (geometric figure) below."

   b. present a word problem describing a geometric figure and require the calculation of the area, perimeter, circumference or volume of that figure. The words area, perimeter, circumference, or volume, need not be explicity stated in the word problem.

2. A test item will require the use of a formula to calculate one of the following:

Calculation	Geometric figure
area	circle rectangle triangle irregular shape (made up of two rectangles or a rectangle and a triangle with the common boundary indicated by a dotted line)
perimeter	polygon
circumference (perimeter)	circle
volume	cylinder rectangular prism

3. Formulas will be provided for all calculations, except for the perimeter of a polygon, area of a rectangle, and area of a triangle. Formulas, as well as an approximate value of π, will be presented in a table in the test booklet.

4. Numbers will be whole numbers and positive decimals. Numbers will not be larger than four digits with two digits to the right of the decimal point.

5. Symbols or abbreviations may be used for measurement units. The symbol ∟ will indicate a right angle.

Answer Choices

1. Each answer choice will be a whole number or positive decimal labeled with the appropriate unit of measure. For items involving π, each answer choice will be presented both in terms of π and solved using $\pi = 3.14$.

2. The correct answer choice will be the value of the requested dimension of the geometric figure.

3. An incorrect answer choice will be a value that is obtained as a result of one of the following types of errors:

   -- **Computation Error**: An error in the addition, subtraction, multiplication, or division of two numbers.

   -- **Regrouping Error**: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.

   -- **Misplaced Decimal Point**: Placing the decimal point in an incorrect position on either a correct solution or an incorrect solution.

   -- **Incorrect Formula**: Not using the correct formula to calculate the dimension of the figure.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because an incorrect formula was used. This answer choice is the circumference ($2\pi r$) of a circle, not the area (πr^2).

2. Answer choice B is also incorrect because an incorrect formula was used. This answer choice is πr, not πr^2.

3. Answer choice C is incorrect because there is a regrouping error in Step 2, as illustrated below.

   Step 1: (no error) Area = $(3.2)^2\pi$

   Step 2: (error)

$$\begin{array}{r} 3.2 \\ \underline{\times\ 3.2} \\ 64 \\ \underline{96} \\ 9.24\ \pi\,\text{cm}^2 \end{array}$$

4. Answer choice D is the correct response, as illustrated below. The correct formula is used and all computations are accurate.

Step 1: $\text{Area} = (3.2)^2\pi$

Step 2:

$$\begin{array}{r} 3.2 \\ \underline{3.2} \\ 64 \\ \underline{96} \\ 10.24\ \pi\ \text{cm}^2 \end{array}$$

Step 3: (if desired) $10.24 \times 3.14 = 32.1536\ \text{cm}^2$

Instructional Emphases

Students may benefit from a brief explanation of the two types of problems encompassed in this objective. Each type requires a somewhat different approach to computing the answer. One type uses a labeled display to present all the information necessary to solve the problem. The second type relies on a word problem to present the information.

For both types of problems, formulas will be provided in a separate table. (The formulas for the perimeter of a polygon, the area of a rectangle, and the area of a triangle will not be provided.) Therefore, students need not memorize any geometric formulas other than those not provided. However, it is important for students to be familiar with the formulas so that they know when each is used. Another essential skill in using these formulas is knowing what each of the terms in the formula means. For example, students must know what a radius is before they can successfully use the formula for the area of a circle. Thus, you may want to review the terminology used in the geometric formulas.

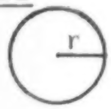
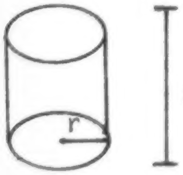
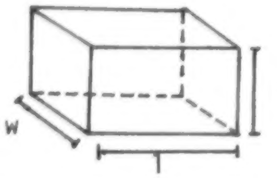
Display problems. It is important that students be able to select the appropriate formula for each problem and correctly substitute the given information into the formula. From that point, students will be required to carry out the computations called for in each formula. (Students need not multiply by a value for π as answers will be presented in terms of π, such as 20.2π.) The key to correctly solving these problems then, is correctly using the appropriate formula for each problem.

Word Problems. Word problems may be more difficult for students to solve. In part this is because not all of the word problems will explicitly state, "Find the area, perimeter, circumference, or volume." Thus, students will have to determine what is being asked by relying on the context of the problem. It may help students to discuss the probable words that are used to describe the terms area, perimeter, circumference, or volume. For example, a problem asking students to determine how much a container will hold, is describing the volume of that container. Once the geometric property is identified, students can then select the appropriate formula to solve the problem. There are several hints that may help students. First, you might encourage students to use sketches when solving these problems. This may help students visualize the geometric figure described in the problem. Students can also incorporate the information presented in the problem in their drawings to further clarify the problem. To do this, students must be familiar with the basic geometric shapes. Students must also be familiar with related terms such as length, width, base, and radius. Knowledge of these terms is essential for students to correctly substitute the given information into the formulas.

Possible Instructional Activities

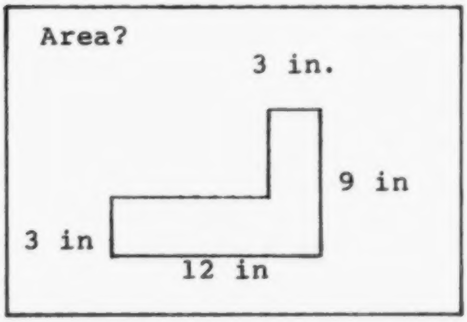
Self-Study

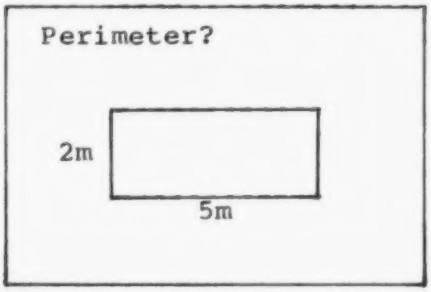
Provide students with the table below.

	Figure	Formula
Circle		Area = πr^2 Circumference (perimeter)= $2\pi r$
Cylinder		Volume = $\pi r^2 h$
Rectangular prism		Volume = lwh

Note: Use $\pi = 3.14$ when computing

Draw pictures of the following geometric figures (circle, rectangle, triangle, irregular shape consisting of two rectangles or a rectangle and a triangle, polygon, cylinder, rectangular prism) on index cards. Provide dimensions (metric and customary) and instructions for each figure. For example:





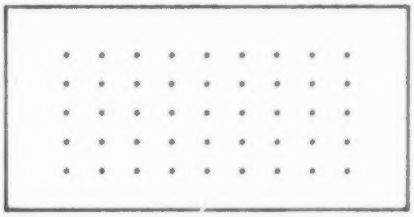
Write the correct answer on the back of each card. The cards may be used for self-study, as follows: A student selects a picture card and then uses the formula table to select the correct formula. Students are responsible for memorizing the formulas that are not included on the formula table (the perimeter of a polygon, area of a rectangle, and the area of a triangle). The student should copy the formula, list the values of the variables, show the substitutions, and solve the problem. Students' answers may be checked by comparing them with the answer written on the back of the picture card.

Search the Area

Have students determine the area of the classroom, their desk, or another area on campus. (Students may determine the area of their homes for more of a challenge, or for extra credit.) You may also add perimeter, circumference, or volume problems of actual objects in the classroom or home.

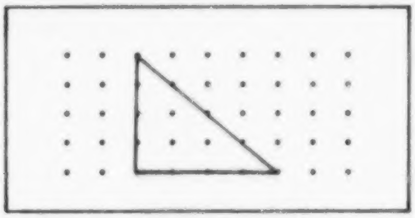
Geoboards

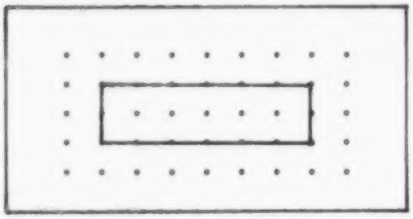
Make a large geoboard or use a commercial geoboard. To make a geoboard, cut a rectangle from plywood (from 8" to 18" sides). Hammer nails into the board, allowing 1" between the nails (horizontally and vertically) and a 1" to 2" border around the board. (See diagram below)



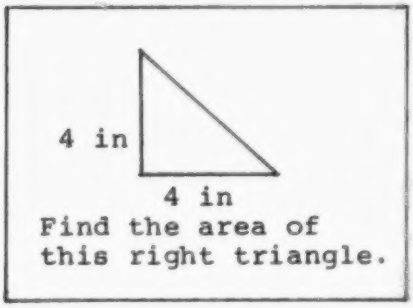
Place the geoboard in a central location in the classroom. Using rubber bands or string, place a new figure on the board each day. Beneath the geoboard have a problem related to the figure. You may even want to put the solution on the back of the problem card so that students may check their answers. Be sure to provide students with any necessary information that they need to solve the problem. Students should solve the problem by the end of the day (or mathematics period). If time permits, have one student demonstrate how to solve the problem.

Example:

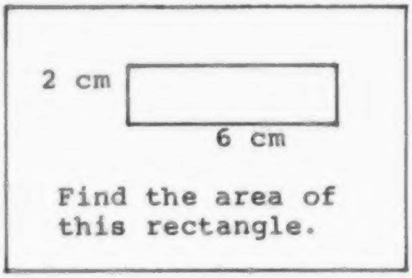




Problem cards



Find the area of this right triangle.



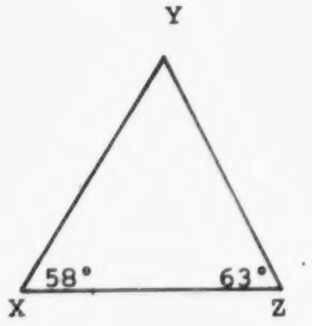
Find the area of this rectangle.

Properties of Geometric Shapes: Use geometric properties to solve problems involving geometric shapes.

This objective requires students to solve for the size of a missing angle or the length of a missing side.

Sample Item

In triangle XYZ, how many degrees are there in angle Y?



A. 69°

B. 239°

C. 59°

D. 159°

Test Questions

1. A test item will require the application of properties of geometric figures to find the length of a missing side or the size of a missing angle.
2. Applications tested will be to:
    a. find the length of a missing side using indirect measurement with similar figures.

    b. find the length of a missing side of a right triangle using the Pythagorean Theorem.

    c. find a missing angle in a triangle using the property that the sum of the angles of a triangle equals 180°.

   d. find a missing complementary or supplementary angle formed by intersecting lines.

3. A test item will present a display labeled with sufficient information to answer the relevant question.

4. Numbers will be whole numbers that are not larger than four digits.

5. The statement of the Pythagorean Theorem will not be provided. Test items which require the student to find the length of a missing side of a right triangle will use numbers that are perfect squares.

6. Symbols or abbreviations may be used for measurement units. The symbol ∟ will indicate a right angle.

Answer Choices

1. Each answer choice for a given test item will be either the number of degrees in a missing angle, or the length of a missing side.

2. The correct answer choice will be the exact solution to the geometry problem presented.

3. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:

   -- Incorrect Property: Use of a geometric property other than the correct one.

   -- Faulty Application: Failing to correctly apply the specified geometric property (computation error).

Answer Analysis for Sample Item

1. Answer choice A is incorrect because there is a faulty application of the required geometric property. This error occurs in Step 1, as illustrated below. Adding 58° to 63° does not equal 111°.

   Step 1: (error) 58° + 63° = 111°

   Step 2: (incorporates error in Step 1) 180° - 111° = 69°

2. Answer choice B is incorrect because an incorrect property was used. The sum of the angles of a triangle is 180° not 360°.

3. Answer choice C is the correct response, as illustrated below. The correct property is used and all computations are accurate.

   Step 1: $58^\circ + 63^\circ = 121^\circ$

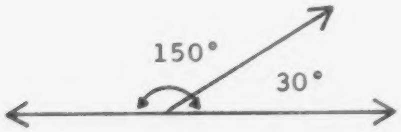
   Step 2: $180^\circ - 121^\circ = 59^\circ$

4. Answer choice D is incorrect because there is a faulty application of the required geometric property. This error occurs when 121° is subtracted from 180° and 159° is obtained.

Instructional Emphases

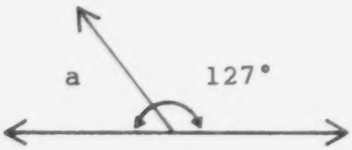
Mastery of this objective requires students to be familiar with several basic "facts" of geometry. It will be helpful to review these facts until students have mastered each one. Each fact, illustrated by a sample problem, is presented below. Students will also be more comfortable with this objective if they have mastered definitions of terms such as angle, right angle, side, and ray.

1. The sum of two angles that form a line equals 180°.



$$\begin{array}{r} 150^\circ \\ +\ 30^\circ \\ \hline 180^\circ \end{array}$$

Students will be presented with problems that require knowledge of this fact such as:



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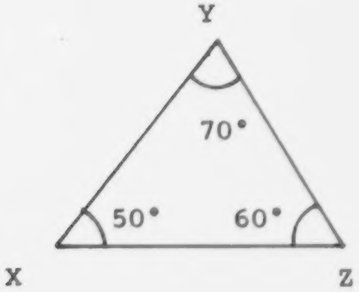
Students will be expected to compute the size of the missing angle. To solve problems like this, students should subtract the angle that is given from 180°.

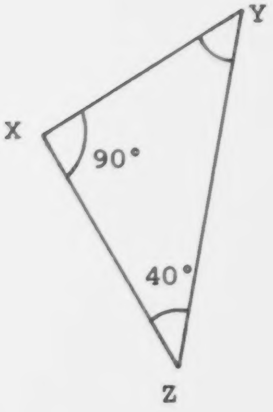
$$\begin{array}{r} 180^\circ \\ -\ 127^\circ \\ \hline 53^\circ \end{array}$$

2. The sum of the three angles that form a triangle equals 180°.



$$\begin{array}{r} 70^\circ \\ 60^\circ \\ +\ 50^\circ \\ \hline 180^\circ \end{array}$$

Problems that involve knowledge of this fact will require students to solve for a missing angle:



To solve for the missing angle students should:

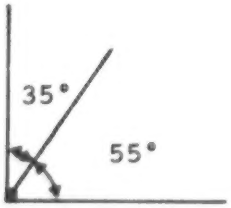
a. Sum the two angles that are given.

$$\begin{array}{r} 90^\circ \\ +\ 40^\circ \\ \hline 130^\circ \end{array}$$

b. Subtract the sum from 180°.

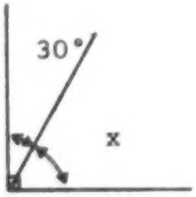
$$\begin{array}{r} 180^\circ \\ -\ 130^\circ \\ \hline 50^\circ \end{array}$$

3. The sum of the angles that bisect a right angle equals 90°.



$$\begin{array}{r} 55^\circ \\ +\ 35^\circ \\ \hline 90^\circ \end{array}$$

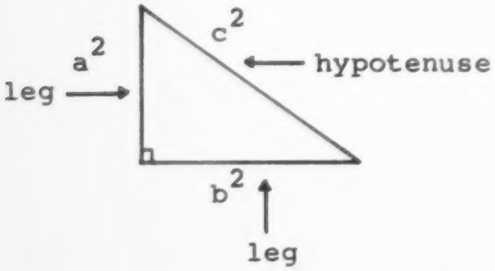
Problems that involve knowledge of this fact will require students to solve for a missing angle:



To solve such a problem students should subtract the angle that is given from 90°. (Make sure students understand that the symbol ∟ indicates a right angle equal to 90°.)

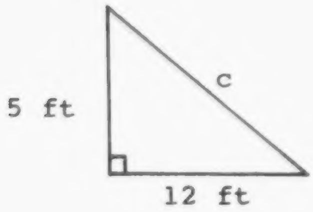
$$\begin{array}{r} 90^\circ \\ -\ 30^\circ \\ \hline 60^\circ \end{array}$$

4. Pythagorean Theorem: In any right triangle the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



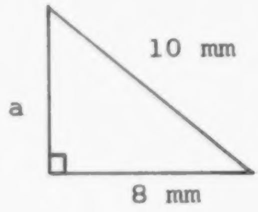
$a^2 + b^2 = c^2$

Problems on the TEAMS test that involve an application of the Pythagorean Theorem will require students to either find the length of a missing leg or the length of the hypotenuse. These problems will present right triangles that have numbers that are perfect squares, thus, the answer will always be a whole number.



To find a missing hypotenuse, students should set up the Pythageorean Theorem and substitute the values of the legs. Then, solve for c.

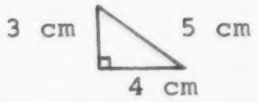
$$\begin{aligned} 5^2 + 12^2 &= c^2 \\ 25 + 144 &= c^2 \\ 169 &= c^2 \\ \sqrt{169} &= c \\ 13 \text{ ft} &= c \end{aligned}$$

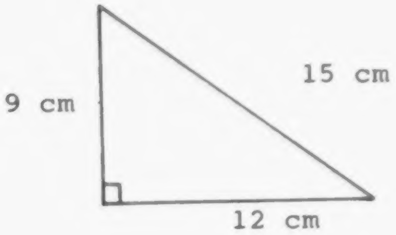


To find a missing leg, students should set up the Pythagorean Theorem and substitute the value of the leg and hypotenuse. Then, solve for a.

$$\begin{aligned} a^2 + 8^2 &= 10^2 \\ a^2 + 64 &= 100 \\ a^2 &= 36 \\ a &= \sqrt{36} \\ a &= 6 \text{ mm} \end{aligned}$$

5. Similar polygons are proportionate in size.





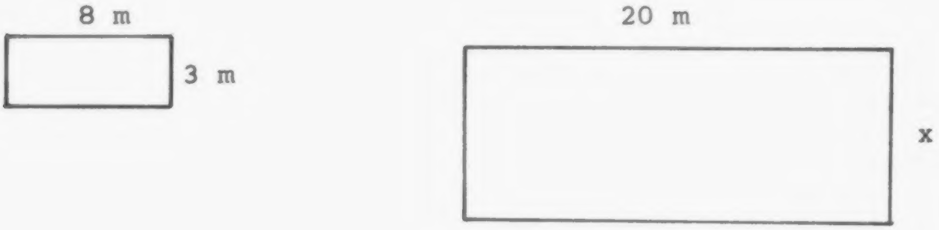
Thus:

$$\frac{3}{9} = \frac{5}{15}$$

$$\frac{3}{9} = \frac{4}{12}$$

$$\frac{4}{12} = \frac{5}{15}$$

Problems that involve knowledge of this fact will require students to solve for a missing side given two similar polygons:



To solve for the length of the missing side students should:

a. Set up a ratio of the two comparable sides (one per figure) that are given:

$$\frac{8}{20}$$

b. Establish an equality that includes a variable for the missing value:

$\frac{8}{20} = \frac{3}{X}$ (8 and 3 are both from the smaller rectangle)
(20 and X are both from the larger rectangle)

c. Cross-multiply and divide to solve for X.

$$\frac{8}{20} \times \frac{3}{X}$$

$$8X = 60$$

$$X = \frac{60}{8}$$

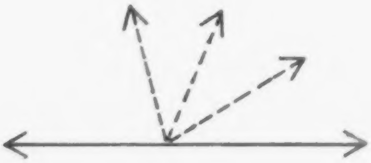
$$X = 7.5 \text{ m}$$

To succeed on this objective, students must know each of the five geometric facts presented above and be able to apply them to solve for the size of a missing angle or the length of a missing side.

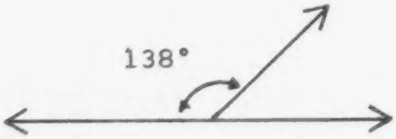
Possible Instructional Activities

Geo-Gram

Give students a copy of a straight line, and ask them to add a ray in any direction.

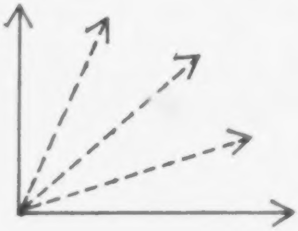


Next, have students measure, using a protractor, one of the angles and write that value on the figure.



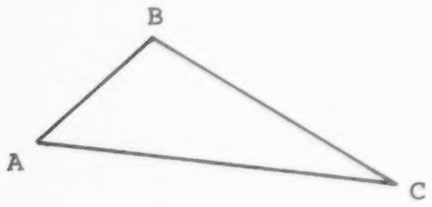
Finally, students should exchange figures and determine the size of the missing angle through knowledge of the geometric "facts." The student who developed the problem can correct it by comparing the answer to that obtained by a protractor.

This exercise can also be used by starting with the figure below and asking students to add a ray to the right angle.



Triangle Trio

Draw a "hypothetical" triangle, any shape, with labels for each vertex.



Then, make a table like that shown below. For each problem, provide the value of two of the angles.

	Angle A	Angle B	Angle C
1.	30°	70°	?
2.	20°	?	90°
3.	?	60°	60°

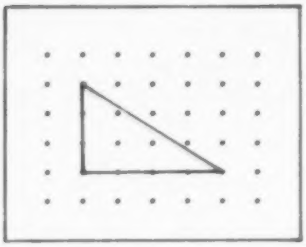
Students must find the size of the missing angle. After the missing angle has been determined, students can construct each triangle using a protractor, if desired.

Geoboard

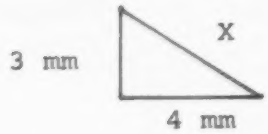
Make a large geoboard or use a commercial geoboard. Directions for making a geoboard are described for the objective on geometric formulas.

Place the geoboard in a central location in the classroom. Using rubber bands, place a right triangle up each day. Beneath the geoboard have a problem related to the figure. The problem should be to either find a missing leg or the hypotenuse. Be sure to use a triangle that forms a perfect square (e.g., 3-4-5, 6-8-10, 5-12-13). You may want to put the solution on the back of the problem card so that students may check their answers. Students should solve the problem by the end of the day (or mathematics period). If time permits, have one student demonstrate how to solve the problem.

Example:



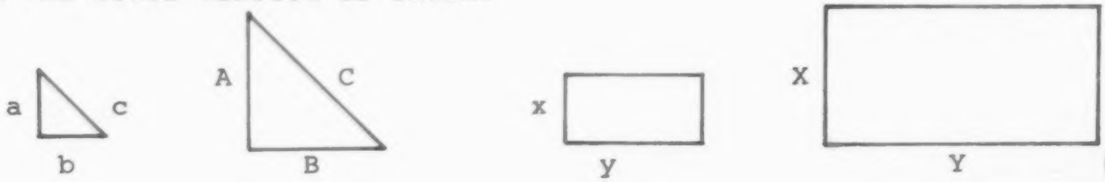
Problem card



Find the length of side X of this right triangle.

Proportion Practice

Draw two similar right triangles or two similar rectangles with the sides labeled as shown:



Then, give three of the missing values (2 on one figure and 1 on the other). For example:

If $a = 5$, $c = 13$ and $A = 10$; $C = ?$	(ans: 26)
If $a = 7$, $b = 8$ and $B = 20$; $A = ?$	(ans: 17.5)
If $x = 12$, $y = 10$ and $X = 30$; $Y = ?$	(ans: 25)
If $X = 40$, $Y = 32$ and $y = 5$; $x = ?$	(ans: 6.25)

Require students to set up equations to solve for the missing values.

Blue-Prints and Negatives

Use actual blue prints or draw a simple blue print of a room. Have students estimate the length or height of the actual building through the use of proportion.

Photo negatives and "blow-ups" can also provide proportion practice. Provide students with the size of a negative and one dimension (length or width) of the finished photograph. Then have students solve for the missing dimension.

SKILL AREA: PROBABILITY, STATISTICS, AND GRAPHING

The skill area of probability, statistics, and graphing covers three objectives: averages, simple probability, and graphs and tables. Students will be presented with word problems for each objective.

Averages: Solve word problems involving averages.

This objective requires the student to solve word problems involving averages of whole numbers or positive decimals.

Sample Item

Century High School's band and choir held a concert series on four evenings. The chart below shows the number of people who attended the four evening concerts. What was the mean (average) number of people attending each night?

Day of the week	Number Attending
Wednesday	108
Thursday	129
Friday	173
Saturday	150

A. 112 people

B. 140 people

C. 560 people

D. 115 people

Test Questions

1. A test item will present a word problem that specifies that the "mean (average)" be calculated for a set of values.

2. A test item will present up to five objects and their values on a specified measure.

3. Numbers will be whole numbers (including 0) or positive decimals. They will not be larger than four digits, with no more than three digits to the right of the decimal point.

Answer Choices

1. The correct answer choice will be an exact solution that is the mean of a set of values.

2. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:

   - Computation Error: An error in the addition, or division of two numbers.

   - Incorrect Operation Error: Performing an operation other than the required one.

   - Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.

   - Missing Operation: Not operating on the data or omitting one of the required operations.

   - Incorrect Divisor: Using an incorrect divisor due to not counting correctly or not counting objects whose value is zero.

   - Misplaced Decimal Point: Placing the decimal point in an incorrect position on either a correct solution or an incorrect solution.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because an incorrect divisor was used. It appears that 560 was divided by 5 instead of 4, yielding the incorrect answer of 112.

2. Answer choice B is the correct response, as illustrated below.

Step 1: (addition)
$$\begin{array}{r}108\\129\\173\\\underline{150}\\560\end{array}$$

Step 2: (division)
$$\begin{array}{r}140\\4\overline{)560}\\\underline{4}\\16\\\underline{16}\\0\end{array}$$

3. Answer choice C is incorrect because an operation is missing. The sum of the people attending each night was not divided by the number of concerts.

4. Answer choice D is incorrect because there is a regrouping error in Step 1, as illustrated below.

Step 1: (error)
$$\begin{array}{r}108\\129\\173\\\underline{150}\\460\end{array}$$

Step 2: (incorporates error in Step 1)
$$\begin{array}{r}115\\4\overline{)460}\\\underline{4}\\6\\\underline{4}\\20\\\underline{20}\end{array}$$

Instructional Emphases

Students are probably quite familiar with the concept of average even though they might not know exactly how to compute one. Grade-point average, batting average, average life-expectancy, and average earnings are common types of averages. Encouraging students to describe averages with which they are familiar may help make this objective more concrete. You may want to review the term "mean" (as this term, along with the term "average," will appear in items measuring this objective). The mean is the sum of all values divided by the number of values. It may be helpful to review how to calculate a mean in terms of specific steps:

Step 1: Sum the values to be averaged.

Step 2: Count the number of values that were added together in Step 1, including zeros.

Step 3: Divide the sum obtained in Step 1 by the total number of values obtained in Step 2.

Because zero may be one of the values to be averaged, you may want to emphasize that zero must be counted as a value during step 2.

Possible Instructional Activities

Average Your Grades

Give students their last three to five test scores and have them compute their average score for the tests. Then, tell students to imagine that they missed the next test and received a zero. Now have students compute the new average and examine the effect of the zero.

On Average, How Much Do You Spend?

Have students keep track of how much of their own money they spend on food each day for a period of four to five days. Have students compute the average amount of money they spend a day for the specified time period. If the students do not spend any of their own money on a particular day, they should record a zero.

What's the Score?

Have students compute the mean score for the home team for the last five or six football (baseball, volleyball, basketball, soccer, or water polo) games.

Interview

As homework, have students interview 8 to 10 friends about a topic such as:

a. the number of hours they watched TV the night before
b. the amount of their allowance
c. the number of brothers and sisters (combined) they have
d. the number of pets at home
e. the number of hours they studied the night before

Then, have students compute the average response to the question(s) they ask.

Simple Probability: Solve word problems involving simple probability.

This objective requires the student to solve word problems involving simple probability.

Sample Item

You have 30 hard candies in a bag. Of them, 8 are cherry flavored, 7 are coconut flavored, 9 are grape flavored, and 6 are raspberry flavored. What is the probability of randomly selecting a raspberry-flavored candy?

A. $\frac{1}{6}$

B. $\frac{3}{10}$

C. $\frac{4}{15}$

D. $\frac{1}{5}$

Test Questions

1. A test item will require the calculation of the probability of an independent event. Both the frequency of the event of interest and the total number of events will be presented in the word problem. Only whole numbers that are not larger than three digits will be used.

Answer Choices

1. Each answer choice will be a positive fraction in simplest form.
2. The correct answer choice will be the probability of the event described in the problem.
3. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:
   - Incorrect Data Error: Performing a necessary operation on the wrong data.
   - Incorrect Simplification: Incorrectly simplifying a fraction.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because the fraction is not simplified correctly. The value $\frac{1}{6}$ is equal to $\frac{5}{30}$ not $\frac{6}{30}$.
2. Answer choice B is incorrect because the wrong data are used to form the fraction. The value $\frac{3}{10}$ is the probability of selecting a grape-flavored candy, not a raspberry-flavored one.
3. Answer choice C is also incorrect because the wrong data are used to form the fraction. This answer represents the probability of choosing a cherry-flavored candy.
4. Answer choice D is the correct response. The correct information is used to form the fraction ($\frac{6}{30}$) and the fraction is simplified correctly.

Instructional Emphases

Explain to students that simple probability allows us to determine how likely it is that a certain event (outcome) will occur. Probability can be estimated by a ratio of the number of times a specified event occurs to the total number of possible events. The probability of an event is always a number from 0 to 1. If an event has probability 0, that means it cannot possibly happen. If an event has probability 1, that means that it must happen. When solving these types of problems, students should use a basic probability ratio.

Probability Ratio:

$$\text{Probability of a particular event} = \frac{\text{number of times a specified event occurs}}{\text{total number of possible events}}$$

Students need to be able to extract the essential probability information from a word problem. Once they have done this, they must be familiar enough with the probability ratio to know where to place the numbers. The following hint may help students with the correct placement of the numbers in the ratio. Namely, the total number of possible events will be the larger number and is always on the bottom of the probability ratio.

You may also want to tell students that the total number of "possible events" can refer to a wide variety of things including objects, occurrences, people, animals, etc. The top half of the probability ratio, which will be the smaller number, refers to the number of times a specified event occurs.

Finally, because the probability of an event will be a fraction, students should be reminded to simplify their answers.

Possible Instructional Activities

Marbles in a Jar

To familiarize students with the differences between probability estimates and actual events, obtain several sets of single-colored marbles, for example, blue, green, white, etc. Place these in a glass jar in known amounts, for instance, 30 blues, 20 greens, 50 reds, etc. Next, having informed students of the numbers of different colored marbles in the jar, ask students to determine the probability of a blindfolded individual selecting, for instance, a blue marble. Then, blindfold one student and have the student select 10 marbles, one at a time (replacing the selected marble in the jar to keep the probabilities constant). Determine whether the estimated probabilities (for example, 3 out of 10) match the actual selections (for example, 1 out of 10). These types of experiments can be repeated with very small numbers of marbles in the jar. Discuss the reasons that probability estimates sometimes fail to coincide exactly with actual events.

The Draw

Use a deck of cards* to help teach this concept. Begin by using the entire deck as the "total number of possible events." Ask students the following questions and have them explain how they arrived at their answers.

---

*Students may not be familiar with a deck of playing cards. Therefore, display a deck of cards showing that a normal pack of 52 cards has four suits--hearts, diamonds, clubs, and spades. Hearts and diamonds are red, while spades and clubs are black. There are 13 cards in each suit: one ace, nine cards numbered 2 to 10, and three "face" cards (the Jack, Queen, and King).

If there are 52 cards in this deck of cards, what is the probability that I will draw (at random):

a. a king of hearts? ($\frac{1}{52}$)

b. an ace of spades? ($\frac{1}{52}$)

c. an ace? ($\frac{4}{52} = \frac{1}{13}$)

d. a red card? ($\frac{26}{52} = \frac{1}{2}$)

e. a diamond? ($\frac{13}{52} = \frac{1}{4}$)

f. a face card? ($\frac{12}{52} = \frac{3}{13}$)

g. a black jack? ($\frac{2}{52} = \frac{1}{26}$)

You might wish to draw several cards from the deck to illustrate the probabilities involved, for example, drawing four cards to see whether one is a diamond (as in question e).

Diamonds Are Forever

Using the same deck of cards, isolate one suit, such as diamonds, and set the remainder of the cards aside. This will change the "total number of possible events" from 52 to 13. (There are 13 cards in each suit.) Ask the following questions:

a. What is the probability that I will draw the ten of diamonds? ($\frac{1}{13}$)

b. What is the probability that I will draw the queen of spades? ($\frac{0}{13} = 0$)

You may vary this activity by isolating certain parts of the deck such as kings only or face cards only to vary the "total number of possible events."

High Rollers

Using a six-sided number cube, you can begin by asking students to identify the total number of possible events (6). Then, ask the following questions:

a. What is the probability of rolling a two? ($\frac{1}{6}$)

b. What is the probability of rolling a number less than three? ($\frac{2}{6} = \frac{1}{3}$)

c. What is the probability of rolling a number less than five? $(\frac{4}{6} = \frac{2}{3})$

d. What is the probability of rolling a number less than or equal to six? $(\frac{6}{6} = 1)$

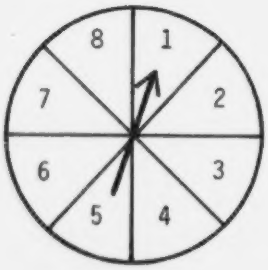
Coin Toss

Ask students about the probability of obtaining "heads" $(\frac{1}{2})$ when tossing a coin. Students can test this prediction by tossing a coin 10 times. Then, have students average the number of "heads" obtained by each student. (The average should be close to 5.)

Spinner

Construct a spinner such as that shown below. Assume that the arrow will not fall on a line. Ask students to answer the following questions:

1. What is the probability the arrow will land on 5? $(\frac{1}{8})$
2. What is the probability the arrow will land on an even number? $(\frac{4}{8} = \frac{1}{2})$
3. What is the probability the arrow will land on an odd number? $(\frac{4}{8} = \frac{1}{2})$
4. What is the probability the arrow will land on a number less than 4? $(\frac{3}{8})$
5. What is the probability the arrow will land on a number divisible by 3? $(\frac{2}{8} = \frac{1}{4})$
6. What is the probability the arrow will land on a prime number? $(\frac{5}{8})$

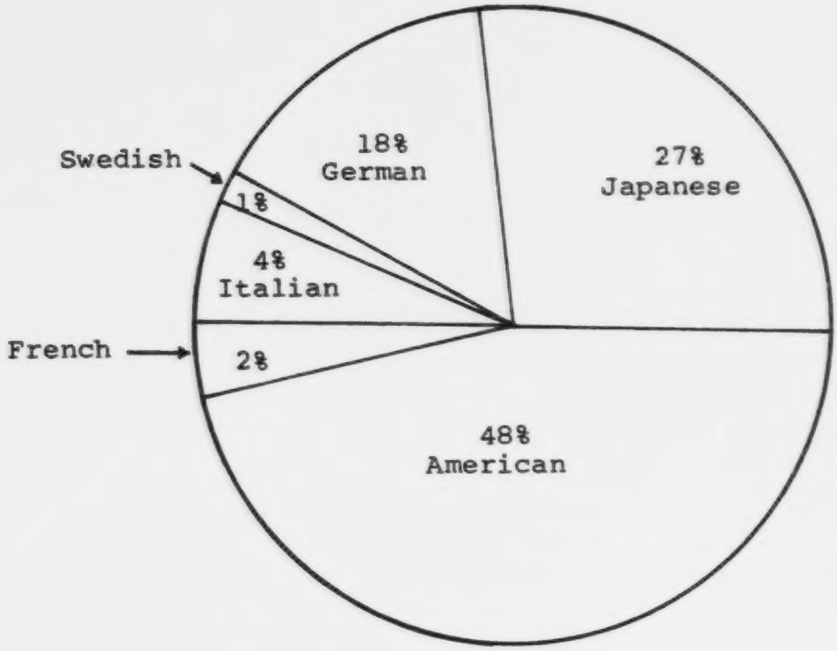


Graphs and Tables: Use information from graphs and tables to solve word problems.

This objective requires the student to solve word problems using information from graphs and tables.

Sample Item

Types of Cars Purchased in the United States



According to this graph, what percent of the cars purchased in the United States are not American?

A. 48%

B. 42%

C. 52%

D. 51%

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Test Questions

1. A test item will present a graph or table and will require the application of one or two operations on data from the display. The operations will be addition, subtraction, multiplication, and division.

2. Numbers from displays will be whole numbers and positive decimals. They will not be larger than four digits with no more than three digits to the right of the decimal point.

3. Addition operations will require the student to add up to five numbers. All other operations will involve only two numbers.

4. For multiplication operations, numbers will not be larger than a four-digit number multiplied by a two-digit number.

5. For division operations, numbers will not be larger than a four-digit number divided by a two-digit number.

6. Displays (graphs and tables) will have the following characteristics:

   a. A bar graph will present one or two relationships and will have no more than 12 bars. The bars may run vertically or horizontally. Up to 10 grid lines perpendicular to the bars may extend across the graph. Students may be required to estimate a graphed value.

   b. A line graph will be horizontal and may present up to three separate lines. Up to 10 horizontal grid lines may extend across the graph. Students may be required to estimate a graphed value.

   c. A circle graph will have no more than eight segments. Each segment will be labeled with a title and a percentage with up to one decimal place.

   d. A table will have between five and twenty numeric entries.

Answer Choices

1. Each answer choice will be a whole number or positive decimal labeled with the appropriate unit from the word problem.

2. The correct answer choice will be the solution to the word problem based on the presented table or graph.

3. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:

   - Computation Error: An error in the addition, subtraction, multiplication or division of two numbers.
   - Incorrect Operation: Performing an operation other than the required one.
   - Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.
   - Missing Operation: Not operating on the data or omitting one of the required operations.
   - Incorrect Order: Performing the required operations in the wrong order.
   - Inaccurate Information: Selecting incorrect information from a table or graph.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because it is missing an operation. 48% was taken directly from the graph and no operations were performed. At least one operation is required to answer a graph and table item.

2. Answer choice B is incorrect because there is a regrouping error. It appears that a 1 was carried from the ones column to the tens column, instead of a 2, yielding the incorrect 42%.

3. Answer choice C is the correct response. You can obtain this answer by subtracting the percent of American cars from 100% (100% - 48% = 52%) or by adding together the percentages of cars that are not American,
27% + 18% + 1% + 4% + 2% = 52%.

4. Answer choice D is incorrect because it contains a computation error when adding the ones column.

Instructional Emphases

This objective requires students to obtain information from a table or graph and perform computations to answer a related question. It may be a good idea to focus your instruction primarily on how to read information from a graph or table, as computation skills will be covered in a variety of other objectives.

To help students learn how to interpret a graph or table, you may want to discuss examples of various types of graphs and tables including the primary purpose of each type of display:

1. Bar Graphs:

   A bar graph typically shows the relative status of data, such as the number of calories in different flavors of ice cream or the gas mileage of various cars. The bars can be compared with each other according to their relative length and can be displayed either vertically or horizontally.

2. Line Graphs:

   A line graph typically shows the change in data, such as rainfall across various years or the number of student absences in various months.

3. Circle Graphs:

   A circle graph typically shows data as percents, such as the percent of time spent studying by subject area or the percent of weekly allowance spent on various items. It is important that students realize that the segments of a circle graph add up to 100 percent.

4. Tables:

   Tables are used to organize and display (in rows and columns) a large amount of numeric information.

An important factor to consider when solving word problems for this objective is that graphs and tables contain both essential and non-essential information. Therefore, it is important for students to pinpoint exactly what each problem is asking and then look for the relevant information on the graph or table.

Possible Instructional Activities

What's in a Graph?

For homework have students find two different types of graphs in newspapers or magazines. For each graph ask students to identify the type of graph and briefly explain the topic.

Pick several of the graphs to discuss with the class. Show them on the board or make copies for students to share. Use the graphs to ask questions that require students to perform one or two computations.

Graph It with Bars

Have students construct a bar graph. For example:

a. Have students keep track of how many minutes of television they watch each day for one school week (five days). Students should summarize the information and make a bar graph to display the summarized data. When constructing the graph, students should use one bar for each day. Be sure students label their graphs.

b. Have students conduct a mini-survey and record the summarized information. One idea would be to survey students about their plans after high-school. Students' responses could be placed in categories such as: College, Work, Work and College, Travel. Students should show the results on a bar graph. They should use one bar to represent each category included in the survey. Be sure students label their graphs carefully. This assignment can be adapted to match the interests of your students. Have students suggest topics for graphing.

c. Have students ask a question related to their graph that requires a computation. Students can exchange graphs and answer the question.

Lines Talk

Have students construct a line graph. For example:

a. Have students record the temperature of a glass of water as it cools. Students can use hot tap water or boiling water to begin. Students should record the temperature every five minutes for a specified number of minutes. They should construct a line graph to show the change in temperature. Have students label the graphs.

b. Have students record the outdoor temperature each hour for a six- to ten-hour period. Students should read the temperature at the same time each hour. Students should construct a line graph to show the change in temperature. Be sure students label their graphs.

c. Have students record the cost of a certain fruit or vegetable each week for at least four weeks. Have students construct a line graph to show the change in price over the specified time period. Students should label the graphs.

d. For all line graphs that are constructed, have a different student devise a practice word problem based on the graph. Have the original developer of the line graph solve the problem. Graphs and problems can be circulated to provide more practice activities for students.

Build-a-Problem

Direct students to locate a table or graph that can be used to construct a problem similar to those used in assessing this TEAMS objective. Be sure that the problem requires one or two operations on data obtained from a graph or table. Select the best of these problems, then copy them to use with the rest of the class for practice exercises. Graphs and tables can also be mounted on index cards with questions on the front and answers on the back. Students can use these self-study cards when they complete other assignments.

SKILL AREA: ALGEBRAIC CONCEPTS

The skill area of algebraic concepts covers two objectives: formulas and solve for a variable. Students will be required to solve word problems using formulas and to solve simple equations.

Formulas: Solve word problems using formulas.

This objective requires the student to solve word problems using formulas. The formulas will require two or three computations to solve for the value of a variable.

Sample Item

Paul plays city league baseball. So far this season he has 50 hits in 225 times at bat. He walked 25 times. Use the formula provided below to calculate his batting average to three decimal places.

$$B = h \div (t - w)$$

where B = batting average
h = hits
t = times at bat
w = walks

A. .200

B. .300

C. .250

D. .025

Test Questions

1. A test item will require the use of a formula to solve for the value of a variable appearing alone on the left-hand side of the formula.
2. A test item will require from one to three values to be substituted into the formula. These values will be provided in the context of the word problem.
3. Variable letters used in the formula will be defined below the formula.

4. Formulas will require either two or three computations. These may include the addition, subtraction, multiplication, or division of whole and decimal numbers, and the addition, subtraction, or multiplication of fractions. Fractions will have a numerator from 1 through 9 and a denominator from 2 through 10. All numbers will be positive and will not be larger than four digits.

Answer Choices

1. The correct answer choice will be the exact solution to the problem presented.
2. An incorrect answer choice will be a number that is obtained as a result of one of the following types of errors:

   -- Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.

   -- Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.

   -- Misplaced Decimal Point: Placing the decimal point in an incorrect position on either a correct solution or an incorrect solution.

   -- Incorrect Operation: Performing an operation other than a required one.

   -- Incorrect Order Operation: Performing an operation outside of parentheses prior to an operation inside of parentheses, or adding/subtracting before multiplying/dividing.

   -- Missing Operation: Not operating on the data or omitting one of the required operations.

   -- Common Denominator Error: An error in selecting the lowest common denominator or in renaming fractions in terms of the lowest common denominator.

   -- Incorrect Simplification: Incorrectly simplifying a fraction.

Answer Analysis for Sample Item

1. Answer choice A is incorrect because a wrong operation was performed, as illustrated on the following page:

Step 1: (no error) $B = 50 \div (225 - 25)$

Step 2: (error: values inside parentheses were added rather than subtracted) $B = 50 \div (250)$

Step 3: (incorporates error from Step 2)

$$\frac{50}{250} = \begin{array}{r} .200 \\ 250\overline{)50.000} \\ \underline{50\ 0} \\ 000 \end{array}$$

2. Answer choice B is incorrect because it contains a computation error in Step 3, as illustrated below:

   Step 3: (division error)

$$\frac{50}{200} = \begin{array}{r} .300 \\ 200\overline{)50.000} \\ \underline{50\ 0} \end{array}$$

3. Answer choice C is the correct response. All values are correctly substituted into the formula, and all computations are accurate.

   Step 1: (substitute values into formula) $B = 50 \div (225 - 25)$

   Step 2: (perform operation inside parentheses: subtraction) $50 \div (200)$

   Step 3: (perform operation outside parentheses: division)

$$\frac{50}{200} = \begin{array}{r} .250 \\ 200\overline{)50.000} \\ \underline{40\ 0} \\ 10\ 00 \\ \underline{10\ 00} \end{array}$$

4. Answer choice D is incorrect due to a misplaced decimal point. Although, the values and computations are accurate, the decimal is incorrectly placed.

Instructional Emphases

For the most part, students will not be familiar with the particular formulas used in formula items. It may be helpful for students to know that familiarity with the formula is not important. What students must know, however, is how to substitute given values into a formula. They should use the definitions of each variable in the formula to help them when substituting values from the word problem. Students must accurately complete the substitution process in order to solve the problem correctly.

Once the values have been substituted properly, students need to solve for the missing variable. The formula makes the "word problem" easier because the appropriate operations are already defined. However, it is essential that students be familiar with the order of operations to be performed in an equation. The basic rules regarding order of operations are discussed below:

a. Perform operations inside parentheses before operations outside of parentheses.

Example: $3 \times (5+3) = \underline{\ ?\ }$

$3 \times 8 = 24$

b. Perform multiplication or division before addition or subtraction.

Example: $3 \times 5 + 3 = \underline{\ ?\ }$

$15 + 3 = 18$ (by comparing this answer to the answer obtained for #1 above, students can be shown that different orders of operations result in different answers)

c. If only multiplication and division are involved, or if only addition and subtraction are involved, the operations are performed from left to right.

Example: $2 \times 6 \div 4 = \underline{\ ?\ }$ Example: $13 - 7 + 2 = \underline{\ ?\ }$

$12 \div 4 = 3$ $6 + 2 = 8$

Using Formulas to Solve Word Problems

It may be helpful to review with students specific steps that they should follow when solving formula problems:

1. Read the problem.

Read the problem to get a general idea of what it is about. Pay particular attention to what values are missing from the formula.

2. Find missing values.

   Search the word problem to find each missing value and substitute it in the appropriate place in the formula.

3. Calculate carefully.

   Perform the calculations in the appropriate order and double-check all work.

4. Is the answer answerable?

   Decide whether the answer seems reasonable. If a problem asks students to compute the outdoor temperature and a student's answer is 320°, that answer is not reasonable given the nature of the question. If an answer does not seem reasonable, check whether a value was substituted in the wrong position or a computation error was made.

Possible Instructional Activities

Weather Report

Have students bring in weather reports from local newspapers. Write these formulas on the board:

$$C = \frac{5(F-32)}{9} \qquad F = \frac{9}{5}C + 32$$

Have the students change reported temperatures from degrees Fahrenheit to degrees Celsius or vice versa by using the formulas above. (It might also be helpful to review the Celsius thermometer with the class.)

How Far?, How Fast?, and How Long?

Using a mathematics textbook, assign students a page on solving word problems using the distance formula. (If such problems are not available in a text, have several students generate such problems for the rest of the class.) Write these three formulas on the board:

$$d = rt, \; r = \frac{d}{t}, \; t = \frac{d}{r}$$

Include definitions of each variable: d = distance, r = rate, and t = time. Explain to students that all three formulas are similar, however, each is used to solve for a different variable.

Interesting Interest

Another formula which may be helpful for practice is $i = prt$, where i = interest, p = principal, r = rate, and t = time of loan. Develop oral or written word problems in which students must substitute values and solve for i.

Moribana

Write these three formulas on the board:

$$H = 1\frac{1}{2} L + D$$

$$P = \frac{2}{3} H$$

$$E = \frac{1}{2} P$$

where:
L = Length of dish
D = Depth of dish

Read the following paragraph to the class:

> "A moribana is a type of Japanese flower arrangement that is made in a shallow dish. It has three main stems which represent heaven (H), people (P), and Earth (E). The appropriate lengths of these stems depend on the length (L) and the depth (D) of the dish in which the arrangement is made. Find the length of each stem (H, P, and E) if the bowl that they will be arranged in is 8 inches long and 3 inches deep."

Have one volunteer come to the board and solve for the "heaven". Have a second volunteer solve for the "people" and a third volunteer solve for the "Earth" stem.

Make a copy of the chart on the following page and give it to students for further practice.

Find the lengths of the stems using the information given:

Dish length L	Dish depth D	Length of the heaven stem(H) $H = 1\frac{1}{2} L + D$	Length of the people stem(P) $P = \frac{2}{3} H$	Length of the Earth stem(E) $E = \frac{1}{2} P$
10 in.	6 in.			
8 in.	2 in.			
6 in.	3 in.			
10 in.	4 in.			

Solve for a Variable: Solve problems to determine the value of a variable.

For this skill students will be presented with simple equations and asked to solve for the value of one variable. The solution will require one or two steps.

Sample Item

Solve for the value of X.

$$X + 6.3 = 66$$

A. 59.7

B. .3

C. 72.3

D. 60.7

Test Questions

1. One or two operations (addition, subtraction, multiplication, or division) will be required to solve for the value of the variable. Only one operation will be required when a fractional coefficient is used.
2. The variable will appear on either the right or the left side, but not both sides, of the equation.
3. Variable coefficients will be whole numbers or fractions with a numerator of one.
4. Numbers for use in the equation will be whole numbers, decimals, and fractions. All numbers will be positive and will not be larger than two digits.

Answer Choices

1. Each answer choice will be a positive whole number, decimal, or fraction. Fractions will be simplified, but will not be written as mixed numbers.
2. The correct answer choice will be the value of the variable that makes the equation true.

3. An incorrect answer choice will be a value that is obtained as a result of one of the following types of errors:

   - Computation Error: An error in the addition, subtraction, multiplication, or division of two numbers.
   - Regrouping Error: Failing to regroup when necessary, regrouping incorrectly, or regrouping unnecessarily.
   - Misplaced Decimal Point: Placing the decimal point in an incorrect position on either a correct solution or an incorrect solution.
   - Missing Operation: Performing one, but not both, of the operations needed to solve for the variable.
   - Incorrect Operation: Performing an operation other than a required one.
   - A Partial Operation: Performing an appropriate operation on some, but not all, of the eligible terms.

Answer Analysis for Sample Item

1. Answer choice A is the correct response. To solve for X you must subtract 6.3 from each side of the equation. Thus, $66 - 6.3 = 59.7$.

2. Answer choice B is incorrect because there is a computation error. More specifically, the decimal points were not properly lined up before subtracting 6.3 from 66, as illustrated below:

$$\begin{array}{r} 66 \\ -\ 6.3 \\ \hline .3 \end{array}$$

3. Answer choice C is incorrect because an incorrect operation was performed. 6.3 was added to, rather than subtracted from, 66.

4. Answer choice D is incorrect because there was a regrouping error in the ones column.

Instructional Emphases

Students must be able to "manipulate" equations to solve for a variable. You will want to stress that the variable must be set alone on one side of the equation before it is possible to determine its value. It is essential that students be familiar with the following Properties of Equality to solve these problems:

Addition Property of Equality

Adding the same number to each side of an equation forms an equivalent equation. For example:

$$X - 6 = 10$$

and

$$X - 6 + 6 = 10 + 6$$

thus

$$X = 16$$

Subtraction Property of Equality

Subtracting the same number from each side of an equation forms an equivalent equation. For example:

$$X + 7 = 20$$

and

$$X + 7 - 7 = 20 - 7$$

thus

$$X = 13$$

Multiplication Property of Equality

Multiplying every term on each side of an equation by the same number forms an equivalent equation. For example:

$$\frac{1}{5}X = 12$$

and

$$5(\frac{1}{5}X) = 12(5)$$

thus

$$X = 60$$

Division Property of Equality

Dividing every term on each side of an equation by the same number forms an equivalent equation. For example:

$$7X = 30$$

and

$$\frac{7X}{7} = \frac{30}{7}$$

thus

$$X = \frac{30}{7}$$

Students will be required to use one or two of these properties to solve each problem. Begin by giving students practice with applying one of these properties, as in the example provided with each property. Then extend this instruction to the use of two properties, such as,

(Subtraction + Division Properties)

$$\begin{aligned} 3X + 5 &= 17 \\ 3X &= 17 - 5 \\ 3X &= 12 \\ X &= \frac{12}{3} \\ X &= 4 \end{aligned}$$

You may also want to show the computations with division first to illustrate that every term on a side must be divided:

$$\begin{aligned} 3X + 5 &= 17 \\ \frac{3X}{3} + \frac{5}{3} &= \frac{17}{3} \\ X &= \frac{17}{3} - \frac{5}{3} \\ X &= \frac{12}{3} \\ X &= 4 \end{aligned}$$

Be sure that students "check" their answer by substituting it into the equation.

Example:

$$\begin{aligned} 3X + 2 &= 12 \\ 3X &= 10 \\ X &= \frac{10}{3} \end{aligned}$$

Check by substituting $\frac{10}{3}$ for X

$$\begin{aligned} \cancel{3}\left(\frac{10}{\cancel{3}}\right) + 2 &= 12 \\ 10 + 2 &= 12 \\ 12 &= 12 \end{aligned}$$

To review, students should:

1. Use the equality properties (one at a time) to set the variable alone on one side of the equation.
2. Do the necessary computations to solve for the variable.
3. If the answer is a fraction, simplify if possible.
4. Finally, check the answer through substitution.

Possible Instructional Activities

Make Your Own

Present the following problems to students. For problem one, read a whole number. Tell students to place the number in one of the two blanks. Then, read a second whole number. Students must place it in the remaining blank. Students should then solve for X and compare answers to see who has the largest solution. Continue through the problems by supplying students with 2 or 3 whole numbers one at a time. Occasionally include a decimal as one of the missing values.

1. $X + \underline{\quad} = \underline{\quad}$
2. $\underline{\quad} = \underline{\quad}X$
3. $X - \underline{\quad} = \underline{\quad}$
4. $\underline{\quad} = \underline{\quad} + X$
5. $\underline{\quad} = \frac{1}{\underline{\quad}}X$
6. $\underline{\quad} = \underline{\quad}X + \underline{\quad}$
7. $\underline{\quad}X - \underline{\quad} = \underline{\quad}$
8. $\underline{\quad} = \underline{\quad} + \underline{\quad}X$
9. $\underline{\quad} = X + \underline{\quad} - \underline{\quad}$
10. $\frac{1}{\underline{\quad}}X = \underline{\quad}$

Which One?

Using a mathematics textbook, select a page of equations requiring students to solve for a single variable. In class have students discuss which Property or Properties of Equality should be used to solve each equation and why. Assign the same problems as homework.

Isolate and Solve

Write several single variable equations on the chalkboard. Have volunteers go to the board and perform the steps necessary to isolate the variable on either side of the equal sign for each equation. Have new volunteers solve each problem for the isolated variable. In the meantime, the remaining students should solve each equation on scratch paper. Have one of the seated students verify that the answers on the board are correct.

Dear Educator:

We hope that this Exit Level Instructional Strategies Guide for Mathematics has provided you with sufficient information and teaching strategies to assist your students with mastering the objectives assessed on the Exit Level Texas Educational Assessment of Minimum Skills test.

We would appreciate your completing this questionnaire so that we may make any needed revisions in this document for future versions.

1. How were you informed about this guide?

   _____ TEA
   _____ ESC
   _____ Superintendent
   _____ Principal
   _____ Curriculum Specialist
   _____ Another Teacher
   _____ Other: Specify ______________________________

2. With how many individuals must you share this document?

   _____ None
   _____ 1-2
   _____ 3-5
   _____ Entire Department
   _____ Other: Specify __________________

3. Is the overall format of this guide easy to follow?

   _____ Yes
   _____ No
   Comments:

4. Were these sections useful for you?

	Yes	No
Description of Objective	_____	_____
Sample Item	_____	_____
Test Questions	_____	_____
Answer Choices	_____	_____
Answer Analysis for Sample Item	_____	_____
Instructional Emphases	_____	_____
Possible Instructional Activities	_____	_____

Comments:

If you would like to suggest additional instructional strategies which may be included in future versions of this document, please describe these below. If you determined that some of our suggested strategies were either inappropriate or did not work well in your class, please list these below so that we may consider deleting them from future versions.

Your response will be greatly appreciated. Please mail it to:

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